MATH 563H MIDTERM REVIEW SPRING 2018

The midterm exam will cover materials from Chapters 2 and 3. It will be based largely on the lectures and homework assignments. More specifically, the following is an outline of the materials.

Please note: you are allowed one sheet of letter size notes for the exam (both sides), but no calculators, phones, textbook, or additional notes.

Chapter 2: Curve theory

1. *Curves in* $\mathbb{R}^n$ *in general:* regular parametrized curves, reparametrization, oriented curves, length of a curve, arc-length parametrization, unit speed parametrization, tangent lines, closed curves, periodic parametrization with period L (and how to determine the period L).

2. *Plane curves:* the definition of curvature of a unit speed parametrized plane curve, normal vector and Frenet frame, Frenet formulae, the geometric meaning of curvature, the formula of curvature for a general regular parametrized curve (not necessarily unit speed), simple closed curves, the winding number of a closed curve, Hopf Theorem, convex curves, vertex of a curve, Four-Vertex Theorem.

3. *Space curves:* the definition of curvature and torsion of a unit speed parametrized space curve, normal vector and binormal vector, Frenet formulae, the formulae of curvature and torsion for a general regular parametrized curve (not necessarily unit speed), total curvature of a closed space curve, the bridge number, the relation between total curvature and the bridge number, simple closed space curves.

Chapter 3: Classical surface theory

Regular surfaces, local parametrizations, some typical examples (regular surfaces as graph of a smooth function, or as a level set of a smooth function), tangent planes and its geometric interpretation, the equation of a tangent plane, the first fundamental form (and how to compute it with respect to a given local parametrization), orientable surfaces, unit normal vector field, oriented surfaces, unit normal vector field for some typical examples (i.e., regular surfaces as graph of a smooth function, or as a level set of a smooth function), the Gauss map, the Weingarten map, how to compute Weingarten map through the definition, the Gauss curvature, the mean curvature, the second fundamental form, the formula for the second fundamental form with respect to a given local parametrization, how to compute Weingarten map through the second fundamental form, principal curvatures, normal curvature, normal curvature and the second fundamental form (Meusnier's Theorem), curvature lines, geometric meaning of the Gauss curvature (i.e., Gauss curvature and the local shape of the surface), integration and area of a surface, area element of a surface, minimal surfaces, mean curvature field, the mean curvature field and the variation of surface area, some standard constructions of surfaces (i.e., ruled surfaces, surfaces of revolution).