MATH 563H FINAL EXAM REVIEW SPRING 2018

The final exam will cover materials from Chapters 4 and 6 only. It will be based largely on the lectures and homework assignments. More specifically, the following is an outline of the materials.

Please note: *you are allowed one sheet of letter size notes for the exam (both sides), but no calculators, phones, textbook, or additional notes.*

Chapter 4: The inner geometry of surfaces

Note: *In the beginning of this chapter, the discussion is focused on the special case where the Riemannian metric is the first fundamental form. The definition of various terms is often done differently, and can not be extended formally to general metrics.*

Smooth vector fields on a surface, smooth functions on a surface, directional derivative of a smooth function, the gradient vector field of a smooth function, Lie bracket of two smooth vector fields, the local formulas of the above items, smooth vector field along a curve and its covariant derivative along the curve, local formula and the Christoffel symbols, properties, covariant derivative of vector fields, properties, second covariant derivative of vector fields, local formulas, covariant derivative and Lie bracket, Riemann curvature tensor, local formula, properties, Theorema Egregium, Riemannian metric, Christoffel symbols, covariant derivative, Riemann curvature tensor and Gauss curvature for a general Riemannian metric, variation of energy, geodesics (including the special case when the metric is the first fundamental form), local form of the geodesic equations, existence and uniqueness of geodesics, flat metric, geodesic curvature, Frenet formula, formula of geodesic curvature for arbitrary parametrization, the exponential map, Riemann normal coordinates, geodesic polar coordinates, Gauss curvature in geodesic polar coordinates, parallel transport, holonomy, and their calculations, Riemann curvature tensor as infinitesimal holonomy, geodesic variation and Jacobi field and some geometric applications, surfaces with constant Gauss curvature, various models for hyperbolic geometry (Poincare disc model and half-plane model), conformally equivalent metrics, conformal mappings, the group of isometries of hyperbolic geometry, the geodesics in hyperbolic geometry, geodesic triangles in spherical and hyperbolic geometry.

Chapter 6: Gauss -Bonnet Theorem

Triangulation of surfaces, Euler characteristic, the Gauss -Bonnet Theorem for geodesic triangles, and the Gauss -Bonnet Theorem.