HOMEWORK 4

SUPPLEMENTARY PROBLEMS

1. Let $S \subset \mathbb{R}^3$ be a regular surface and let g be a Riemannian metric on S. Investigate how the Christoffel Symbols Γ_{ij}^k, and Riemann curvature tensor R_{ijk}^l, and the Gauss curvature K change under the following change of g, i.e., replacing g by the scalar multiple $r \cdot g$, where $r > 0$ is a constant.

2. Let $S \subset \mathbb{R}^3$ be a regular surface and let g be a Riemannian metric on S. Let (U, F, V) be a local parametrization. We define for $i, j = 1, 2$,

$$g_{ij}(u) = g_{F(u)}(\frac{\partial F}{\partial u^i}, \frac{\partial F}{\partial u^j}), \text{ where } u \in U.$$

Let R_{ijk}^l be defined by the formula $R(\frac{\partial F}{\partial u^i}, \frac{\partial F}{\partial u^j})\frac{\partial F}{\partial u^k} = \sum_{l=1}^2 R_{ijk}^l \frac{\partial F}{\partial u^l}$. Then prove that the Gauss curvature K satisfies

$$K = \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 g^{jk} R_{ijk}^i,$$

where (g^{ij}) is the inverse of the 2×2 matrix (g_{ij}).