HOMEWORK 3

SUPPLEMENTARY PROBLEMS

1. Let $S \subset \mathbb{R}^3$ be the graph of $z = 1 - x^2 - y^2$, and let f be the smooth function on S which is the restriction of the smooth function $f(x, y, z) = z$ defined on $\mathbb{R}^3$. Recall the gradient vector field of f, grad f, is the smooth vector field on S defined by the following equation: for any vector field X on S,

$$\partial_X f = I(\text{grad } f, X),$$

where I is the first fundamental form of S. Use the parametrization

$$F(x, y) = (x, y, 1 - x^2 - y^2)$$

of the surface S to give a concrete formula for grad f, and show that grad f is equal to the orthogonal projection of the gradient vector of the function $f(x, y, z) = z$ in $\mathbb{R}^3$ to the tangent planes of S.

2. Let $S \subset \mathbb{R}^3$ be the graph of the function $f(x, y) = 1 - x^2 - y^2$. We consider the parametrization $F(r, \theta) = (r\cos\theta, r\sin\theta, 1 - r^2)$, where $r > 0$, of S.

(1) Compute the Christoffel symbols Γ_{ij}^k from its definition, for $i = 1$, $j = 2$.

(2) Compute the Christoffel symbols Γ_{ij}^k using the formula which depends only on the first fundamental form of S, for $i = 1$, $j = 2$.

(Remark: you should get the same answers!)