

HOMEWORK 1

SUPPLEMENTARY PROBLEMS

1. Consider plane curve $c(t) = r(t) \cdot (\cos t, \sin t)$, where $r(t)$ is given by

(1) $r(t) = 2 + \cos(2t)$,

(2) $r(t) = 2 + \cos(t/2)$.

For the two cases (1) and (2) as above, answer the following questions:

- Show that $c(t)$ is periodic and find its period.
- Is $c(t)$ a simple closed curve?
- Compute the curvature function $k(t)$.
- What is the winding number n_c ?
- Is the curve $c(t)$ convex?
- Find all the vertices of the curve $c(t)$.
- Sketch the curve $c(t)$.

2. Consider the space curve (called the helix) $c(t) = (a \cos t, a \sin t, bt)$ where $a, b > 0$ are constants. Compute the curvature $k(t)$ and torsion $\tau(t)$ of the curve $c(t)$.