Course Outline for
MATH 456: MATHEMATICAL MODELING
Spring 2016

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Webpage: Link to the course page through my personal webpage at www.math.umass.edu/~turk
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Office Hours: Tues. 2:30—4:00, Wed. 2:30—4:00, and by appointment
Lectures: MWF 11:15—12:05 in LGRC A201 (Lederle Lowrise)
Teaching Assistant: Jonathan Maack **Office:** LGRT 1323N. **Hours:** TBA.

Prerequisites for this course.

Calculus I, II, III, as in Math 131, 132, 233; that is, a thorough knowledge of differential and integral calculus in one, two and three real variables. Linear Algebra and Differential Equations, as in Math 235 and 331.

Course objectives and approach.

The term "modeling" has an extremely wide range of meaning. At its most general, modeling can be viewed as any cognitive activity that uses symbols to understand, or mentally grasp, observed phenomena. In this philoshopical sense our minds continually build models of the world around us to represent our knowledge of reality. "Mathematical modeling" elevates this natural process of all rational thought to a method of scientific investigation. To do so, it focuses on those phenomena or questions that are amenable to a quantitative description, and it aims mainly at those models which produce definite predictions or support clear decisions.

We will learn how to formulate, analyze and apply mathematical models. Our goal will be to use the mathematics that you already know (everything from high school algebra and geometry through calculus, linear algebra and differential equations) to ask and answer questions that occur in science, engineering or any other real-world context. To do so requires that we also rely on whatever general knowledge of science you have acquired in your education.

Given the nature of mathematical modeling, our approach will be different from that taken in standard mathematics classes and textbooks, which tend to concentrate on deductive systems of reasoning and techniques of calculation within a particular field. Our investigations will be driven by scientific questions rather than mathematical theories. We will be willing to adopt any method as long as it yields useful results.

A successful mathematical model provides an accurate and reliable representation of the phenomenon chosen for study. But there is no unique degree of model faithfulness. Most interesting real phenomena are composite and complex. Part of the act of modeling is to decide on what to include and what to ignore. Different models can be formulated to give different insights into the same general phenomenon, or a hierarchy of models can be created to capture behavior with increasing levels of detail. Thus, modeling is an inherently open-ended and creative process.

The lectures will present some classic models that are remarkably effective and serve as examples or paradigms for modeling in general. But most models are far from perfect — the phenomenon or question under investigation may not translate well into mathematical terms, the equations governing the model may be too difficult to solve, or the results derived from the model may not agree with experiments or observations. Rather than be discouraged by this problematic side of modeling, we will learn that even a successful model has its limitations. To be useful and informative, a model does not need to be perfect or comprehensive.

Integrative Experience.

Math 456 is one of the Integrated Experience courses approved by the General Education Council. Every major must complete an IE course as a graduation requirement. These courses must integrate knowledge from the major field with general knowledge gained in courses outside the major, and they must have an element that involves open-ended inquiry as well as group interactions and communications. These University requirements largely dictate the structure of Math 456.

Because of the IE requirement, we must give priority for registration to Mathematics and Statistics majors, with first priority to seniors who need it for graduation. Every effort to accommodate these students will be made, as long as the class size is manageable. But the interactive nature of the course, and the need to make time for group presentations, means that the class size should not exceed 36 students.

Readings and source material.

There is no assigned textbook for this course. Since the course endeavors to apply what mathematics majors have learned in other courses, and the range of possible applications is endless, any one book would be too limited. The students will be responsible for gathering information from the literature as needed in their investigations. The instructor and the teaching assistant will provide guidance when necessary.

Grading procedure.

- **Homework Assignments, totaling 50% of grade.**
  These assignments will consist of short modeling exercises. Since these exercises are not merely technical questions ("solve this equation by that method") such as typically appear in textbooks, you should leave yourself enough time for pondering and revising. The homework problems will be assigned regularly, with 2 weeks for completion.

- **Group Modeling Project, worth 40% of grade.**
  Each student will join a 3-person group and the group will investigate a modeling problem of their choice. The instructor will facilitate the formation of these groups, as well as the selection of suitable topics; but the students in each group will originate their topical question and their modeling approach. The topic may be from any field, as long as it allows the student group to pursue a mathematical modeling project. There will be three required outcomes:
  (1) A 10-minute group progress report at the mid-semester point, describing the topic chosen, the goal of the modeling, and how the group will proceed to construct and analyze their model.
  (2) A 25-minute group final presentation near the end of the semester; this will be a prepared lecture to the class given with projector slides. It will summarize the modeling project – what motivated the questions under investigation, how they are translated into mathematical terms, how the results and conclusions are derived, and whatever open questions remain.
  (3) A final paper that reports the group modeling project. Each student will write and submit a personal version of the project report, even though it reports on joint work. Each student's report will include a separate, one-page, "reflection" on his/her shared learning experience. This reflection should answer questions such as: What were your main contributions to the group project? Were you able to draw on special knowledge that you acquired in the past? Did your ideas, proposals or criticisms become incorporated into the report and presentation? How did the interaction among group members influence the development of the model? Are you satisfied with the final result and with the group experience, including the contributions of the other members of your group?

- **Classroom Participation, worth 10% of grade.**
  Attendance is mandatory throughout the semester for the following reasons.
  (1) The syllabus is wide-ranging, and the lectures will be taken from a variety of source material, not otherwise available from a particular textbook or website.
  (2) All students much be fully engaged members of their assigned groups.
  (3) When the various groups make their progress reports or final presentations, all other students in the class must be present to listen, ask questions and benefit from the interaction.