COURSE OUTLINE FOR MATH 425 — Spring 2009

Instructor: Professor Bruce Turkington
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Webpage: Link to course page from www.math.umass.edu
Office: LGRT 1423K
Office Hours: Tues. 2:00—4:00, Wed. 2:00—4:00, and by appointment
Lectures: Mon., Wed., Fri. 10:10 – 11:00 in LGRT 121
Text: *Vector Calculus, 5th edition*, by J. E. Marsden and A. J. Tromba. (W. H. Freeman and Co.)

Prerequisites for this course: Calculus I, II, III. That is, a basic knowledge and understanding of differential and integral calculus in one, two and three real variables.

Core topics covered in this course:

We will follow the textbook fairly closely, and we will try to cover most of the topics in the book. Only Chapters 1 and 5 will be treated lightly and quickly, because they refer to material that is essentially part of the courses prerequisites (basic calculus as in Math 233).

The lectures will develop all the main concepts and techniques in

- Chapters 2, 3 — Differential calculus in several variables
- Chapter 4 — Vector functions and vector fields
- Chapter 5, 6 — Integral calculus in several variables
- Chapter 7,8 — Vector analysis in two and three dimensions

Applications to be discussed as time permits:

The concepts and techniques in this subject are best appreciated by considering their applications in science. Some of the applications that we will use to motivate and illustrate the mathematical theory include:

- Optimization — First and second order conditions; Lagrange multipliers and constraints
- Basic geometry of curves and surfaces — Tangent, normal and binormal to a space curve; tangent plane and normal vector to a surface
- Fluid motion — Velocity fields; physical interpretation of divergence and curl in terms of flux and circulation; basic conservation laws of fluid mechanics

Course objectives and approach:

This is a course on "advanced calculus," which is the bedrock of all mathematical analysis. The subject matter is very similar to what a freshman or sophomore learns in the first three semesters of calculus, but now the emphasis is on understanding the general case and dealing with it in a more sophisticated way. Accordingly, we consider functions from the space R^n of n real variables, where $n = 1, 2, 3, \ldots$. And we allow these functions to take values in the space R^m, where $m = 1, 2, 3, \ldots$. Thus, we develop the calculus of arbitrary smooth functions $f : R^n \to R^m$.

Given the generality of this subject matter, it is important to keep some concrete applications in mind. In our study of vector-valued functions of one variable, $f : R^1 \to R^n$, we will think about paths or curves, while for functions scalar-valued functions of many variables, $f : R^n \to R^1$, we will study potentials or cost functions. Throughout we will endeavor to develop an intuition about these functions and the calculus objects related to them (their derivatives or integrals, whatever is appropriate). For this reason, we will intersperse of development of the mathematics with applications to science. The textbook does this also. Our approach to the mathematics will be through systematic derivations and careful calculations, but we will not emphasize rigorous proofs. In that way we can get through the key elements of the theory and to some significant applications in one semester.

Grading procedure:

- **Final Exam [120 minutes]:** 30%
  Time and place: As dictated by University Exam Schedule for Spring 2009.
- **2 Midterm Exams [50 minutes]:** 20% each
  Dates:– Mid1 on Wednesday, February 25; Mid2 on Wednesday, April 8
- **Graded homework [weekly]:** Totaling 30%
  Assigned on Monday of each week, due on the following Monday.

Homework:

Homework problems will be assigned weekly, which you will hand in on Monday class following the Monday that these problems are assigned. All or some of the assigned problems will be graded, depending on the available time of the grader. Of course, it is crucial to do all these problems yourself in order to learn the material.

Read ahead and come to class:

Always keep up to date with your reading and re-reading of the textbook. It should be clear which section we are going to discuss each day of class, and you are strongly encouraged to read that section *before* the class. That way you will get the most out of our presentation and discussion. Then shortly *after* class you should re-read the section in the textbook *and* your lecture notes to reinforce your understanding.