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# INVARIANT LINEAR CONNECTIONS ON HOMOGENEOUS SYMPLECTIC VARIETIES

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**Abstract.** We find all homogeneous symplectic varieties of connected semisimple Lie groups that admit an invariant linear connection.

## Introduction

Let $G$ be a connected Lie group. One can consider the following problem: to describe all homogeneous $G$-spaces $X$ that admit an invariant linear connection (see [2], [3], [5]). Not much is known in general and the main attention was focused on the description of invariant linear connections on some particularly nice homogeneous spaces, mainly on reductive homogeneous spaces (see [1], [2]). The aim of this paper is to give a complete solution of this problem in the following situation: $G$ is a connected semisimple Lie group over field $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ and $X \cong G/H$ is a symplectic $G$-variety, that is, $X$ is either an adjoint orbit of $G$ or its covering. Our answer is given by the following theorem:

**Theorem 1.** *Let $X$ cover the adjoint orbit $\mathrm{Ad}(G)x$, $x \in \mathfrak{g}$. Let $x = x_s + x_n$ be the Jordan decomposition in $\mathfrak{g}$, $\mathfrak{z}(x_s) = \mathfrak{z} \oplus \mathfrak{g}_1 \oplus \ldots \oplus \mathfrak{g}_m$ the decomposition into a sum of the center $\mathfrak{z}$ and simple ideals $\mathfrak{g}_k$ of the centralizer of $x_s$, and one has $x_n = x_n^1 + \ldots + x_n^m$, where $x_n^k \in \mathfrak{g}_k$, $k = 1, \ldots, m$. Then $X$ admits an invariant linear connection if and only if, for $k = 1, \ldots, m$,*

$$x_k \neq 0 \Rightarrow \mathfrak{g}_k \cong \mathfrak{sp}(2n_k, \mathbb{K}),\ n_k \in \mathbb{N} \text{ and } x_k \text{ is a highest root vector in } \mathfrak{g}_k^{\mathbb{C}}. \tag{1}$$

It is known [3] that semisimple adjoint orbits always admit an invariant linear connection and the situation of nilpotent orbit is one of the most important. In §1 we deduce Theorem 1 from the following particular cases.

**Theorem 2.** *Let $G$ be a connected simple complex Lie group, and let $X$ be a nonzero nilpotent adjoint $G$-orbit. The following assertions are equivalent:*

(a) *there exists an invariant linear connection on $X$;*
(b) *there exists an invariant linear connection on some covering $\widetilde{X}$ of $X$;*
(c) *$\mathfrak{g} \cong \mathfrak{sp}_n(\mathbb{C})$ and $X = (\mathrm{Ad}\, G)e$, where $e$ is a highest root vector.*

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**Theorem 3.** *Let $G$ be a connected simple real Lie group, and let $X$ be some covering of a nonzero nilpotent adjoint orbit $\mathrm{Ad}(G)e$, $e \in \mathfrak{g}$. Then $X$ admits an invariant linear connection if and only if $\mathfrak{g} \cong \mathfrak{sp}_n(\mathbb{R})$ and $e$ is the highest root vector in $\mathfrak{g}^{\mathbb{C}} = \mathfrak{sp}_n(\mathbb{C})$.*

A connection is called *symplectic* if the corresponding covariant derivative of the basic symplectic form $\omega$ vanishes on $X$. It is observed in [7, §6], that for any $G$-invariant linear connection $\nabla$ on $X$ without torsion there exists a canonically defined symplectic $G$-invariant linear connection $\overline{\nabla}$ also without torsion. Namely, $\overline{\nabla}$ differs from $\nabla$ by a symmetric tensor $T(\xi,\eta) = \overline{\nabla}_\xi(\eta) - \nabla_\xi(\eta)$ of type $(1,2)$ defined by

$$\omega(T(\xi,\eta),\zeta) = \frac{1}{3}((\nabla_\xi\omega)(\eta,\zeta) - (\nabla_\eta\omega)(\zeta,\xi)), \tag{2}$$

for any vector fields $\xi$, $\eta$, $\zeta$ on $X$.

Our result can be applied to studying $*$-products on $X$ and formal deformations of the Poisson Lie algebra on $X$ (see. [6]–[9]). In particular, it is known ([9]) that $X$ admits a $G$-invariant $*$-product if and only if there exists a symplectic $G$-invariant linear connection on $X$, i.e., only in the cases described in Theorem 1.

The paper is organized as follows. In §1 we recall some basics on invariant linear connections, and deduce Theorems 1 and 3 from Theorem 2. In §2 we prove the Basic Lemma, which says that if there exists an invariant linear connection on a nilpotent orbit $X$, then $X$ is locally $G$-equivariantly isomorphic to an open orbit in some $G$-module. The Basic Lemma is proved a priori using an $\mathfrak{sl}_2$-trick. In §3 we prove Theorem 2 using the well-known list of all $G$-modules having an open orbit.

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## §1. Basics on invariant linear connections

Let $G$ be a connected Lie group over $\mathbb{R}$ or $\mathbb{C}$ with Lie algebra $\mathfrak{g}$. Let $X$ be a homogeneous $G$-variety. Denote by $H = G_o \subset G$ the stabilizer of some point $o \in X$, and let $\mathfrak{h}$ be the Lie algebra of $H$. The tangent space $T_oX$ is canonically identified with $\mathfrak{g}/\mathfrak{h}$. For any $\xi \in \mathfrak{g}$ we denote the vector $\xi + \mathfrak{h} \in \mathfrak{g}/\mathfrak{h}$ by $\overline{\xi}$. If $A : \mathfrak{g} \to \mathfrak{g}$ is a linear operator such that $A\mathfrak{h} \subset \mathfrak{h}$, then $\overline{A}$ is the induced operator on $\mathfrak{g}/\mathfrak{h}$, $\overline{A}(\overline{\xi}) = \overline{A\xi}$. Any $\xi \in \mathfrak{g}$ defines the vector field $\xi^*$ on $X$. The corresponding map $\mathfrak{g} \to \mathrm{Vect}(X)$ is a Lie algebra homomorphism. Since $X$ is homogeneous, the values of the vector fields $\xi^*$ at any point of $X$ span the tangent space at that point. In particular, we have $\xi^*(o) = \overline{\xi}$.

It is known (see [3, Theorem 2] or [5]) that there exists a 1-1 correspondence between the set of invariant linear connections on $X$ and the set of linear maps $\Gamma : \mathfrak{g} \to \mathrm{End}\,\mathfrak{g}/\mathfrak{h}$ that satisfy the following conditions for any $\xi \in \mathfrak{g}$, $\alpha \in \mathfrak{h}$, $x \in H$:

$$\Gamma(\alpha) = \overline{\mathrm{ad}(\alpha)} \tag{3}$$

$$\Gamma(\mathrm{Ad}(x)\xi) = \overline{\mathrm{Ad}(x)}\Gamma(\xi)\overline{\mathrm{Ad}(x)}^{-1}. \tag{4}$$

If $H$ is connected, then (4) is equivalent to

$$\Gamma([\alpha,\xi]) = [\Gamma(\alpha),\Gamma(\xi)]. \tag{5}$$

In fact, the conditions (3), (5) are equivalent to the existence of an invariant linear connection on the universal covering of $G/H$.

The covariant derivative of the vector field $\xi^*$ in the direction $\eta^*(o) = \overline{\eta}$ is given by

$$\nabla_{\overline{\eta}}\xi^*(o) = \Gamma(\xi)\overline{\eta}. \tag{6}$$

A connection is called *locally flat* if it has zero curvature and torsion. The curvature $\rho$ and the torsion $\tau$ of the connection $\nabla$ at the point $o$ are given by

$$\begin{aligned} \rho(\overline{\xi},\overline{\eta}) &= \Gamma([\xi,\eta]) - [\Gamma(\xi),\Gamma(\eta)], \\ \tau(\overline{\xi},\overline{\eta}) &= \Gamma(\xi)\overline{\eta} - \Gamma(\eta)\overline{\xi} - \overline{[\xi,\eta]}. \end{aligned} \tag{7}$$

Let us deduce Theorem 3 from Theorem 2.

*Proof of Theorem 3.* If $X$ admits an invariant linear connection, then the system of linear equations (3) and (5) has a solution. It follows from Theorem 2 that the system is consistent only if $\mathfrak{g}^{\mathbb{C}} \cong \mathfrak{sp}_n(\mathbb{C})$ and $\mathfrak{h}^{\mathbb{C}}$ is the centralizer of a highest root vector.

On the other hand, $\mathfrak{h}$ is the centralizer of some element $e \in \mathfrak{g}$. Hence, $\mathfrak{h}^{\mathbb{C}}$ is the centralizer of $e$ in $\mathfrak{g}^{\mathbb{C}}$ and so $e$ is a highest root vector in $\mathfrak{g}^{\mathbb{C}}$. But $\mathfrak{sp}_n(\mathbb{R})$ is the unique real form of $\mathfrak{sp}_n(\mathbb{C})$ containing such vector.

Conversely, let $\mathfrak{g} \cong \mathfrak{sp}_n(\mathbb{R})$ and $e$ is a highest root vector in $\mathfrak{g}^{\mathbb{C}}$. Then $X$ covers the orbit $\mathrm{Ad}(G)e \cong (\mathbb{R}^n \setminus \{0\})/\{\pm 1\}$ that admits a natural $G$-invariant linear connection. □

Since the space $X = G/H$ is symplectic, one has the moment map $\mu : X \to \mathfrak{g}$ (we identify $\mathfrak{g}^*$ and $\mathfrak{g}$ by means of the Cartan inner product) and the covering map $X \to \mathrm{Ad}(G)x$, $x = \mu(o) \in \mathfrak{g}$, where $o = eH$. Let $x = x_s + x_n$ be the Jordan decomposition of $x$ in $\mathfrak{g}$, where $x_s$ is semisimple, $x_n$ is nilpotent and $[x_s, x_n] = 0$. Since $G$ is reductive, the centralizer $G_1 = Z_G(x_s) \subset G$ of $x_s$ in $G$ is a connected reductive subgroup. One has $x_n \in \mathfrak{g}_1$ and $\mathfrak{h} = \mathfrak{g}_1 \bigcap \mathfrak{z}_{\mathfrak{g}}(x_n)$.

**Lemma 1.** *The space* $\mathrm{Ad}(G)x$ *(resp. its covering* $X$*) admits a* $G$*-invariant linear connection if and only if there exists a* $G_1$*-invariant linear connection on* $\mathrm{Ad}(G_1)x_n$ *(resp. on its covering* $X_1 = G_1/H$*).*

*Proof.* The assertion follows from the decomposition $\mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{m}$, where $\mathrm{Ad}(G_1)\mathfrak{m} \subset \mathfrak{m}$, $\mathfrak{h} \subset \mathfrak{g}_1$ and Theorem 4 of [3]. □

*Proof of Theorem* 1. Due to Lemma 1 we may assume that $x = x_n$ is nilpotent. Suppose that $X$ admits an invariant linear connection. The corresponding map $\Gamma$ satisfies conditions (3) and (5). Denote $\mathfrak{h}_i = \mathfrak{h} \bigcap \mathfrak{g}_i$. Consider the map $\Gamma_i : \mathfrak{g}_i \to \mathrm{End}(\mathfrak{g}_i/\mathfrak{h}_i)$ defined by $\Gamma_i(\xi)\overline{\eta} = \mathrm{pr}_i \Gamma(\xi)\overline{\eta}$ for $\xi \in \mathfrak{g}_i$, $\overline{\eta} \in \mathfrak{g}_i/\mathfrak{h}_i$ where $\mathrm{pr}_i : \mathfrak{g}/\mathfrak{h} \to \mathfrak{g}_i/\mathfrak{h}_i$ is the projecting map along $\bigoplus_{j \neq i} \mathfrak{g}_j/\mathfrak{h}_j$. It is easy to see that $\Gamma_i$ gives rise to an invariant linear connection on the universal covering of the space $X_i = \mathrm{Ad}(G)x_i$. Actually, $X_i$ is a nilpotent adjoint orbit of the simple Lie group $G_i \subset \mathrm{Ad}(G)$ with the tangent algebra $\mathfrak{g}_i$. Now (1) follows from Theorems 2 and 3.

Let (1) hold. Then by Theorems 2 and 3 there exist invariant linear connections on $\mathrm{Ad}(G_i)x_i$. The direct sum of the corresponding maps $\Gamma$ satisfies conditions (3), (4) and hence gives rise to an invariant linear connection on $\mathrm{Ad}(G)x$. Since $X$ covers $\mathrm{Ad}(G)x$, it also admits an invariant linear connection. □

From now on we assume that $G$ is a complex Lie group. It is important that we can average connections in the following sense:

**Lemma 2.** *Suppose that $K \subset G$ is a reductive subgroup that normalizes $H$ and $X$ admits a $G$-invariant linear connection $\nabla$. Then $X$ admits an invariant linear connection $\overline{\nabla}$ such that the corresponding map $\Gamma$ satisfies the following additional condition:*

$$\Gamma(\mathrm{Ad}(x)\xi) = \overline{\mathrm{Ad}(x)}\Gamma(\xi)\overline{\mathrm{Ad}(x)}^{-1} \text{ for any } x \in K,\ \xi \in \mathfrak{g}.$$

*Proof.* We note that conditions (3) and (4) define an *affine* subspace $M$ in the linear space $L = \mathrm{Hom}(\mathfrak{g}, \mathrm{End}\,\mathfrak{g}/\mathfrak{h})$. The group $K$ acts on $M$ in a natural way. If a reductive group $K$ acts linearly on a vector space $L$ and preserves an affine subspace $M$, then $M$ must contain a $K$-fixed vector that represents the desired connection. □

The lemma's assertion means that $\overline{\nabla}$ is invariant under the $G$-equivariant action of $K$ on $G/H$ by right shifts.

## §2. Invariant linear connections on nilpotent orbits

Let $e \in \mathfrak{g}$ be a nilpotent element, $X = \mathrm{Ad}(G)e$, and let $\widetilde{X}$ be the universal covering of $X$. By the Jacobson–Morozov Theorem the nilpotent element $e \in \mathfrak{g}$ can be included into the $\mathfrak{sl}_2$-triplet $(f, h, e) \subset \mathfrak{g}$.

**Basic Lemma.** *Suppose that there exists an invariant linear connection on $\widetilde{X}$. Then there exists the unique invariant linear connection on $\widetilde{X}$ such that the corresponding map $\Gamma$ satisfies the following additional condition:*

$$\Gamma([h, \xi]) = [\overline{\mathrm{ad}\, h}, \Gamma(\xi)] \text{ for any } \xi \in \mathfrak{g}. \tag{8}$$

*This connection is locally flat and symplectic.*

*Proof.* Let $T$ be the one-parameter subgroup whose Lie algebra is spanned by $h$. Clearly $T$ normalizes $H = G_e$ and hence its identity component $H^0$. By Lemma 2 we obtain (8) for some connection on $\widetilde{X}$.

**Lemma 3.** *There exists an $\mathfrak{sl}_2$-triplet $\langle \mathcal{F}, \mathcal{H}, \mathcal{E} \rangle \subset \mathrm{End}\,\mathfrak{g}/\mathfrak{h}$, such that $\mathcal{E} = \overline{\mathrm{ad}\, e}$ and $\mathcal{H} = \overline{\mathrm{ad}\, h} + \mathrm{Id}_{\mathfrak{g}/\mathfrak{h}}$.*

*Proof.* Let $Y \subset \mathfrak{g}$ be an irreducible $\mathfrak{sl}_2$-submodule in $\mathfrak{g}$ with highest weight $\mu$ and highest weight vector $y$. Then $\mathfrak{y} \equiv Y \bigcap \mathfrak{h} = \langle y \rangle$. The operator $\mathrm{ad}\, e|_Y$ induces the unique structure of a simple $\mathfrak{sl}_2$-module on $\overline{Y} \equiv Y/\mathfrak{y}$, the highest weight of which is $\mu - 1$ and the highest vector is $\overline{[f, y]}$. We assume by definition that the desired $\mathfrak{sl}_2$-triplet acts on $\overline{Y}$ in accordance with described structure. We notice that if $[h, \xi] = k\xi$ for $\xi \in Y$, then $\mathcal{H}|_{\overline{Y}}\overline{\xi} = (k+1)\overline{\xi}$; therefore $\mathcal{H} = \overline{\mathrm{ad}\, h} + \mathrm{Id}_{\mathfrak{g}/\mathfrak{h}}$. □

It follows from Lemma 3, equation (8), and (5) that for any $\xi \in \mathfrak{g}$ we have

$$\Gamma([e, \xi]) = [\mathcal{E}, \Gamma(\xi)], \quad \Gamma([h, \xi]) = [\mathcal{H}, \Gamma(\xi)] \quad \text{for any } \xi \in \mathfrak{g}. \tag{9}$$

It is well known that for any reductive group $S$ and any $S$-modules $U$ and $V$ the linear map $U \to V$ is $S$-equivariant if and only if it is equivariant with respect to a Borel subgroup of $S$. It follows that the equations (9) imply that we also have

$$\Gamma([f,\xi]) = [\mathcal{F}, \Gamma(\xi)] \text{ for any } \xi \in \mathfrak{g}. \tag{10}$$

Substituting $e$ and $h$ instead of $\xi$ in (10) we get $\mathcal{H} = \Gamma(h)$ and $\mathcal{F} = \Gamma(f)$. Therefore, for any $\xi \in \mathfrak{g}$

$$\Gamma([f,\xi]) = [\Gamma(f), \Gamma(\xi)], \quad \Gamma([h,\xi]) = [\Gamma(h), \Gamma(\xi)], \quad \Gamma([e,\xi]) = [\Gamma(e), \Gamma(\xi)]. \tag{11}$$

The subalgebras $\mathfrak{h} \subset \mathfrak{g}$ and $\langle f, h, e\rangle \subset \mathfrak{g}$ together generate $\mathfrak{g}$. Therefore, (5) and (11) imply that

$$\Gamma([\xi,\eta]) = [\Gamma(\xi), \Gamma(\eta)] \text{ for any } \xi, \eta \in \mathfrak{g}. \tag{12}$$

So $\Gamma$ is a uniquely defined representation of $\mathfrak{g}$ in $\mathfrak{g}/\mathfrak{h}$ and the curvature of the corresponding invariant linear connection $\nabla$ vanishes by (7).

We are to check now that $\nabla$ is torsion-free. Let us consider the symmetrization of $\nabla$, which is an invariant linear connection without torsion. Obviously, it satisfies (8) and consequently it coincides with $\nabla$, so $\nabla$ is locally flat. Furthermore, consider the symplectic connection $\overline{\nabla}$ defined by (2). It follows from (6) that $\overline{\nabla}$ satisfies (8). Hence $\nabla = \overline{\nabla}$ is a symplectic connection. The Basic Lemma is proved. □

## §3. Proof of Theorem 2

*Proof.* The implication (a) $\Rightarrow$ (b) is trivial since any invariant linear connection on $X$ can be lifted to an invariant linear connection on $\widetilde{X}$.

Let us prove that (c) $\Rightarrow$ (a). Let $G \cong Sp_n(\mathbb{C})$. The adjoint representation of $G$ is isomorphic to the symmetric square of the standard representation of $G$. This isomorphism takes the orbit of the highest root vector to the variety of nonzero "perfect squares" $\{u^2 \in \mathrm{S}^2\,\mathbb{C}^n \mid u \in \mathbb{C}^n \backslash \{0\}\}$. The open orbit in the standard representation of $G$ in $\mathbb{C}^n$ covers this orbit. Therefore, $X$ is obtained from $\mathbb{C}^n \setminus \{0\}$ with a natural action of $G$ by identifying opposite points. Obviously, this variety admits an invariant linear connection.

Finally let us check that (b) $\Rightarrow$ (c). It follows from (b) that there exists an invariant linear connection on the universal covering $\widetilde{X}$. Moreover, we can assume that this connection satisfies the assertions of the Basic Lemma.

It follows from [3, §5] that the invariant linear connection $\nabla$ given by the linear map $\Gamma : \mathfrak{g} \to \operatorname{End} \mathfrak{g}/\mathfrak{h}$ is locally flat if and only if the map $\Gamma$ is a locally transitive representation of $\mathfrak{g}$ in $\mathfrak{g}/\mathfrak{h}$, that is, if there exists a vector $v \in \mathfrak{g}/\mathfrak{h}$ such that $\Gamma(\mathfrak{g})v = \mathfrak{g}/\mathfrak{h}$. Furthermore, it follows from [3, §6] that all locally transitive representations of simple algebras $\mathfrak{g}$ are given in the following table:

| | $\mathfrak{g}$ | | $\Gamma$ |
|---|---|---|---|
| 1 | $\mathrm{A}_l$, | $l \geq 1$ | $kR(\pi_1)$, $k = 1, 2, \ldots, l$ |
| 2 | $\mathrm{A}_{2l}$, | $l \geq 2$ | $R(\pi_2)$ |
| 3 | $\mathrm{A}_{2l}$, | $l \geq 2$ | $R(\pi_2) \oplus R(\pi_2)$ |
| 4 | $\mathrm{A}_{2l}$, | $l \geq 2$ | $R(\pi_1) \oplus R(\pi_2)^*$ |
| 5 | $\mathrm{C}_l$, | $l \geq 2$ | $R(\pi_1)$ |
| 6 | $\mathrm{D}_5$ | | $R(\pi_4)$ |

Here $\pi_k$ is the $k$-th fundamental weight of $\mathfrak{g}$ (in the numbering of [4]). $R(\lambda)$ is an irreducible representation of $\mathfrak{g}$ with the highest weight $\lambda$, and $R(\lambda)^*$ is its dual. $R_1 \oplus R_2$ is the direct sum of $R_1$ and $R_2$, $kR$ is the direct sum of $k$ copies of $R$.

Note that since the connection $\nabla$ is symplectic by the Basic Lemma, the corresponding representation $\Gamma$ is also symplectic. So, case 5 is the only possible one.

The adjoint representation of a simple group has a unique nonzero orbit of minimal dimension, namely, the orbit of a highest root vector. Therefore in the case 5 the only possible variant was considered above. The theorem is proved. □

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