Math 233H Fall 2008. Practice Midterm-2.

The practice midterm contains more problems than the actual test. Midterm problems will be of the same difficulty level but they will be new and original. This means that it is not enough to learn how to solve practice midterm problems to pass the test. A good grasp of concepts and ability to solve similar problems is required. Many people find solving odd-numbered problems in the textbook a nice way to prepare (they all have answers at the end of the book). In fact, most of the problems in this practice midterm are also taken from the book.

1. Let $f(x,y) = \ln(x^2+y^2)$ and consider the point $P = (1,2)$. (a) Find the gradient of f at P. (b) Find the maximal rate of change of $f(x,y)$ at P and the direction at which it occurs. (c) Find the directional derivative of $f(x,y)$ at P in the direction of the polar angle $\pi/6$. (d) Find the direction at which the directional derivative of $f(x,y)$ at P is equal to $4/5$.

2. (a) Find equation of the tangent plane to the surface

$$x^2 - 2y^2 + z^2 = 2 - yz$$

at the point $(2,1,-1)$. (b) At what point on the paraboloid $y = x^2 + z^2$ is the tangent plane parallel to the plane $x + 2y + 3z = 4$?

3. Find the local minimum and maximum values and saddle points of the function (a) $x^3 - 12xy + 8y^3$; (b) $y^2 - 2y\cos x$, $1 \le x \le 7$.

4. Use Lagrange multipliers to find the maximum and minimum values of the function xyz subject to the constraint $x^2 + 2y^2 + 3z^2 = 6$.

5. Find the absolute maximum and minimum values of

$$f(x,y) = 2x^2 + 3y^2 - 4x - 5$$

in the region $x^2 + y^2 \le 16$.

6. Find an equation of the plane that passes through the point $(1,2,3)$ and cuts off the smallest volume in the first octant.

7. (a) Use a Riemann sum with $m = n = 2$ to estimate the value $\iint_R \sin(x+y)\,dA$, where $R = [0, \frac{2\pi}{3}] \times [0, \frac{2\pi}{3}]$ (use the Midpoint rule). (b) Compute this double integral expicitly. (c) Compute $\iint_R xye^{x^2y}\,dA$, where $R = [0,1] \times [0,2]$.

8. (a) Compute $\iint_R y^3\,dA$, where R is the triangular region with vertices $(0,2)$, $(1,1)$, $(3,2$. (b) Evaluate the integral $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx$ by reversing the order of integration.

9. Find the volume of the solid bounded by cylinders $z = x^2$, $y = x^2$, and the planes $z = 0$, $y = 4$.