Math 233H Fall 2008. Practice Midterm-1.

1. (a) Consider the line L through points $A = (2, 1, 1)$ and $B = (5, 3, 2)$. Find the intersection of the line L and the plane given by $2x - 3y + 4z = 13$. (b) Find the distance of the point $(2, 1, 1)$ and the plane given by $2x - 3y + 4z = 13$. (c) Consider the parallelogram with vertices A, B, C, D such that B and C are adjacent to A. If $A = (3, 5, 1)$, $B = (5, 1, 4)$, $D = (5, 2, 3)$, find the point C.

2. Consider the points $A = (2, 1, 0)$, $B = (1, 0, 2)$ and $C = (0, 2, 1)$. (a) Find the vector projection of the vector $\vec{AC}$ onto the vector $\vec{AB}$. (b) Find the distance from the point C to the line L that contains points A and B.

3. (a) Find paramteric equations for the line of intersection of the planes $3x+2y-z = 4$ and $2x+z = 1$. (b) Let L_1 denote the line through the points $(1, 0, 1)$ and $(1, 4, 1)$ and let L_2 denote the line through the points $(2, 3, 1)$ and $(4, 4, 3)$. Do the lines L_1 and L_2 intersect? If not, are they skew or parallel?

4. (a) Find the volume of the parallelepiped such that the following four points $A = (1, 4, 2)$, $B = (3, 1, 2)$, $C = (4, 3, 3)$, $D = (1, 0, 1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A. (b) Find an equation of the plane through A, B, D. (c) Find the angle between the plane through A, B, C and the xy plane.

5. The velocity vector of a particle equals $\vec{v}(t) = 2t\vec{i} + 2\sqrt{t}\vec{j} + \vec{k}$ at any time $t \geq 0$. (a) At the time $t = 0$ this particle is at the point $(1, 5, 4)$. Find the position vector $\vec{r}(t)$ of the particle at the time $t = 4$. (b) Find an equation of the tangent line to the curve at the time $t = 4$. (c) Does the particle ever pass through the point $P = (80, 41, 13)$? (d) Find the length of the arc traveled from time $t = 1$ to time $t = 2$. (e) Find the oscillating plane to the curve at time $t = 4$.

6. Consider the function $f(x, y) = 6x^3y/(2x^4 + y^4)$. (a) Does the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Why or why not? (b) Compute the second partial derivatives of $f(x, y)$ and verify by calculation that $f_{xy} = f_{yx}$. (c) Is this function differentiable at the point $x = 1$, $y = 1$? Why or why not? (d) Write down the equation of the tangent plane to the graph $z = f(x, y)$ at the point $(1, 1, 2)$.

7. Consider the quadric surface $x^2 - 3y^2 - z^2 = 3$. (a) Sketch it and give its name. (b) Using implicit differentiation, compute z_x and z_y at the point $(4, 2, 1)$. (c) Compute the differential dz at the point $(4, 2, 1)$ and use it to approximate the value of z at $x = 4.1$, $y = 1.8$.