STAT 515 - Statistics I
Midterm Exam One

Name: ______________________________
Section: ______________________________

Instruction:

- Notes, calculators, and other electronics are not allowed.
- Show all your work. You need to fully justify all of your answers. A correct answer with insufficient justification may receive no credit.
- A formula sheet is provided in the last page.
- You have 2 hours to complete the exam. There are 6 problems in the exam.

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Good Luck !

Problem	Points
1	/17
2	/18
3	/12
4	/18
5	/20
6	/15
Total	/100

1. A pair of dice are rolled. Each die has the shape of a polyhedron with four sides numbered $1, 2, 3, 4$.

   (a) (4 point) List all the sample points for this experiment.

   (b) (4 points) Let A be the event that exactly one 4 is observed and B the event that at least one 3 is observed. List the sample points in A and B.

   (c) (9 points) Find the following probabilities:

      (i) $P(\overline{A} \cup B)$.

      (ii) $P(A \cap B)$.

      (iii) $P(A|\overline{B})$.

2. (a) (6 points) Three companies produce colored balls. 20% of balls from company A are red, 30% of balls from company B are red, and 70% of balls from company C are red. You buy a ball from a store (which buys its balls with equal probability from A, B, or C) and observe that it is not red. What is the conditional probability that the store bought the ball from company C.

   (b) (6 points) 100 balls are purchased, 20 blue, 50 red, and 30 white. 10 balls are drawn at random from these 100 balls. What is the probability that three out of these 10 are red (you can leave your answer in combinatorial form)?

   (c) (6 points) In the context of part (b), what are the expected value and variance for the number of red balls that are drawn?

3. (12 points) Let Y be a random variable with distribution $p(y)$ given by $p(1) = 0.4$, $p(2) = 0$, $p(3) = 0.2$, $p(4) = 0.1$, $p(5) = 0.3$, and $p(y) = 0$ if $y \neq 0, 1, 2, 3, 4, 5$. Compute the following (you can leave your final answer for each part as a sum):

   (a) $\mathrm{E}(Y)$.

   (b) $\mathrm{Var}(Y)$.

   (c) $\mathrm{E}(Y^3 + 3)$.

4. (a) (6 points) A company produces colored balls, and colors 20% blue, 50% red, and 30% white. Twenty balls are purchased at random. What is the probability that at least 15 of them are red (you can leave your answer in combinatorial form)?

   (b) (6 points) A person buys balls from the company (without knowing the color in advance) until the ball that he buys is red. What is the probability that he buys exactly 3 balls (you can leave your answer in combinatorial form)?

   (c) (6 points) In part (b), if each ball costs 2 dollars, what is the expected amount of money the person spends on colored balls.

5. The eruptions of a certain volcano occur according to a Poisson process with an average of 3 eruptions every year. A scientist observes the volcano for a period of 10 years.

   (a) (6 point) Let the random variable X be the total number of eruptions observed by the scientist. Compute $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

   (b) (7 points) Find the probability that the scientist observes 2 or more eruptions over this period of 10 years.

   (c) (7 points) A documentary film maker films this volcano every day of one year and makes a documentary movie with the footage. If the film maker's profit is given as

   $$10000Y^3 - 100$$

   dollars, where Y is the number of eruptions that year, compute the expected profit of the film maker. Hint: The moment generating function can be helpful.

6. (a) (6 points) Suppose the moment generating function of a random variable Y is given by $m(t)$. Show that the moment-generating function of $X = 4Y + 6$ is $e^{6t}m(4t)$.

(b) (3 points) Suppose $\mathrm{E}(Y) = 2$, $\mathrm{E}(Y^2) = 5$, and $\mathrm{E}(Y^3) = 10$. Compute $\mathrm{E}(X^3)$.

(c) 6 points) Under the assumptions of the previous parts, use Chebyshev's inequality to find a lower bound for $P(-1 < Y < 5)$. That is, find the largest positive number $c > 0$ such that $P(-1 < Y < 5) \geq c$.

Forumla Sheet

The following formulas may be useful:

- $P_k^n = \dfrac{n!}{(n-k)!}$, $\quad C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$.
- Binomial: $p(y) = \binom{n}{y} p^y (1-p)^{n-y}$, $\mu = np$, $\sigma^2 = np(1-p)$, $m(t) = (pe^t + (1-p))^n$.
- Geometric: $p(y) = p(1-p)^{y-1}$, $\mu = \frac{1}{p}$, $\sigma^2 = \frac{1-p}{p^2}$, $m(t) = \frac{pe^t}{1-(1-p)e^t}$.
- Negative binomial: $p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$, $\mu = \frac{r}{p}$, $\sigma^2 = \frac{r(1-p)}{p^2}$.
- Hypergeometric: $p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$, $\mu = \frac{nr}{N}$, $\sigma^2 = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$.
- Poisson: $p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$, $\mu = \lambda$, $\sigma^2 = \lambda$. Moment generating function $m(t) = e^{\lambda(e^t - 1)}$.