Statistics I Midterm Exam One

Name: _____

Instruction:

- You can use a calculator that has no connection to the Internet. Books, notes, cellphones, computers are NOT allowed in the test.
- Please circle eight problems that you want to be graded. Otherwise I will grade the first eight problems. Each problem is worth 15 points. If you do extra ones, each of them is worth 3 points.
- The exam is 2 hours. **Good Luck !**

1 2 3 4 5 6 7 8 9 10

1 A population of voters contains 40% of Republicans and 60% of Democrats. It is reported that 30% of the Republicans and 70% of the Democrats favor an election issue. A person chosen at random from this population is found to favor the issue in question. Find the conditional probability that this person is a Democrat.

2 Suppose two balanced coins are tossed and the upper faces are observed.

a List the sample points for this experiment

b Assign a reasonable probability to each sample point

c Let A denote the event that exactly one head is observed and B the event that at least one head is observed. List the sample points in both A and B

d From your answer to part (c), find $\mathbb{P}(A)$, $\mathbb{P}(B)$, $\mathbb{P}(A \cap B)$, $\mathbb{P}(A \cup B)$, and $\mathbb{P}(\bar{A} \cup B)$.

3 If A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.2$, find the following:

a $\mathbb{P}(A \cup B)$

b $\mathbb{P}(\bar{A} \cap \bar{B})$

c $\mathbb{P}(\bar{A} \cup \bar{B})$

4 If two events, A and B, are such that $\mathbb{P}(A) = 0.3$, $\mathbb{P}(B) = 0.3$, and $\mathbb{P}(A \cap B) = 0.1$, find the following:

a $\mathbb{P}(A|B)$
b $\mathbb{P}(B|A)$
c $\mathbb{P}(A|A \cup B)$
d $\mathbb{P}(A|A \cap B)$
e $\mathbb{P}(A \cap B|A \cup B)$

5 A student prepares for an exam by studying a list on ten problems. She can solve six of them. For the exam, the instructor selects five problems at random from the ten on the list given to the students. What is the probability that the student can solve all five problems on the exam?

6 A particular sale involves for items randomly selected from a large lot that is known to contain 10% defectives. Let Y denote the number of defectives among the four sold. The purchaser of the items will return the defectives for repair, and the repair cost is given by $C = 3Y^2 + Y + 2$. Find the expected repair cost.

7 Assume an HIV test is 99% correct for people with HIV and 99% for people without HIV. Assume in a country 0.3% of the population have HIV. If a randomly selected people tests positive, what is the probability that he has HIV?

8 If Y has a geometric distribution with success probability p.

a Show that for a positive integer n,

$$\mathbb{P}(Y > n) = (1-p)^n$$

b Show that for positive integers n and m

$$\mathbb{P}(Y > m+n \,|\, Y > n) = (1-p)^m = \mathbb{P}(Y > m)\,.$$

9 If A and B are independent events, show that A and $\bar{B}$ are also independent.

10 Consider the experiment, called the *birthday problem*, where our task is to determine the probability that in a group of people of a certain size there are a least two people who have the same birthday (the same month and day of month). Suppose there is a room with 40 people in it, find the probability that at least two people have the same birthday. (Show all your steps. You don't need to compute the exact number.)

(Scratch paper)

Forumla sheet

Laws of probability:

- Additive law:

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

- Multiplicative law:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B|A) = \mathbb{P}(B)\mathbb{P}(A|B)$$

- Law of total probability:
  Let $\{B_1, \cdots, B_N\}$ be a partition of S such that $\mathbb{P}(B_i) > 0$. Then

$$\mathbb{P}(A) = \sum_{i=1}^{N} \mathbb{P}(A|B_i)\mathbb{P}(B_i)\,.$$

- Baye's law

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A|B)\mathbb{P}(B)}{\mathbb{P}(A)}\,.$$

Binomial distribution $B(N, p)$:

- Probability distribution

$$p(n) = \binom{N}{n} p^n (1-p)^{N-n}$$

- Mean Np
- Variance $Np(1-p)$.

Geometric distribution

- Probability distribution

$$p(n) = p(1-p)^{n-1}$$

- Mean $1/p$
- Variance

$$\frac{1-p}{p^2}\,.$$