STAT 515 **MIDTERM 2 EXAM** **November 16, 2016**

In problems that require reasoning, algebraic calculation, or the use of your graphing calculator, it is not sufficient just to write the answers. You must explain how you arrived at your answers and show all your algebraic calculations.

1. As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable Y with a density function given by

$$f(y) = \frac{b}{y^2}, \quad y \geq b \quad \text{and} \quad f(y) = 0, \quad \text{otherwise}.$$

where b is the minimum possible time needed to traverse the maze.

(a) Show that $f(y)$ has the properties of a probability density function.

(b) Find $F(y) = P(Y \leq y)$.

(c) Find $P(Y > b + c)$ for a positive constant c.

(d) If c and d are both positive constants such that $d > c$, find $P(Y > b + d \,|\, Y > b + c)$.

2. The SAT and ACT college entrance exams are taken by thousands of students each year. The mathematics portions of each of these exams produce scores that are approximately normally distributed. In recent years, SAT mathematics exam scores have averaged 480 with standard deviation 100. The average and standard deviation for ACT mathematics scores are 18 and 6, respectively.

(a) An engineering school sets 550 as the minimum SAT math score for new students. What percentage of students will score below 550 in a typical year?

(b) What score should the engineering school set on the ACT math test in order to achieve the same cut-off percentage as the SAT math test in part (a)?

(c) What is the percentage of students with SAT scores between 280 and 680?

(d) Compare the answer to (c) (which is based on the normal distribution) to the answer you get when you use Tchebysheff's Theorem, by explaining why the results may differ.

3. The random variables X and Y denote the lengths of life, in hundreds of hours, for components of types I (for X) and II (for Y), respectively, in an electronic system. Their joint density is given by

$$f(x, y) = e^{-(x+y)}, \quad \text{if} \quad x > 0, y > 0 \qquad \text{and} \qquad f(x, y) = 0 \quad \text{otherwise}.$$

(a) Find the $f_X(x)$ and $f_Y(y)$ marginal densities.

(b) What is the conditional density of X given that $Y = y$?

(c) Are the X and Y random variables dependent or independent? Justify your answer.

(d) Find the probability that a component of type II will have a life length in excess of 200 hours.

4. Consider the probability density function

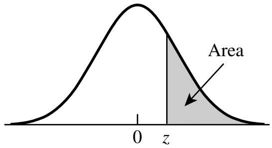
$$f(x,y) = 6(1-y)\,, \quad 0 \le x \le y \le 1 \qquad \text{and} \qquad f(x,y) = 0 \quad \text{otherwise}\,.$$

(a) Find the covariance of X and Y, $\mathrm{Cov}(X,Y)$.

(b) Are the X and Y random variables dependent or independent? Justify your answer.

(c) Find the conditional density $f_{Y|X}(y\,|\,x)$.

Table 4 Normal Curve Areas
Standard normal probability in right-hand tail
(for negative values of *z*, areas are found by symmetry)



	Second decimal place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 003 40									
5.0	.000 000 287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).