

In problems that require reasoning, algebraic calculation, or the use of your graphing calculator, it is not sufficient just to write the answers. You must explain how you arrived at your answers, show your algebraic calculations.

1. Students attending a particular state university can select from 130 major areas of study. A student's major is identified in the registrar's records with a two- or three-letter code. Some students opt for a double major and complete the requirements for both of the major areas before graduation. The registrar was asked to consider assigning these double majors a distinct two- or three-letter code so that they could be identified through the student records' system.
 - (a) What is the maximum number of possible double majors available to the students?
 - (b) If any two- or three-letter code is available to identify majors or double majors, how many codes are available?
 - (c) How many major codes are required to identify students who have either a single major or a double major? Are there enough major codes available to identify all single and double majors at the state university?

2. (a) If A and B are two mutually exclusive events and $P(B) > 0$, show that

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}.$$

(b) If A and B are independent sets, show that A and \bar{B} are also independent. Hint: You may also use a Venn diagram to facilitate your calculations.

(c) Suppose A and B are two events such that $P(A) = .8$ and $P(B) = .7$. Is it possible that $P(A \cap B) = .5$? Is it possible that $P(A \cap B) = .75$? Explain your answers fully, and give examples if you respond "yes" to any of the questions.

3. A diagnostic test for a disease is such that it correctly detects the disease in 90% of the individuals who actually have the disease. Furthermore, if the person does not have the disease, the test will come out negative with probability .9. Only 1% of the population has this disease. If a person is chosen at random, what is the probability that the test will give a "false negative", i.e. the probability the person has the disease given that the test came out negative. What is the probability of a "true negative" result? Hint: Use the Bayes' Rule.

4. An oil exploration firm has raised enough capital to carry out 10 explorations, while the probability of success of each one is 10% based on historic data.

(a) Assuming that the explorations are independent, find the probability that at least 3 out of the 10 will be successful.

(b) What is the probability this firm will fail to find oil before running out of cash?

(c) The same firm would like to know what is the probability it will strike oil for the first time within the first five attempts.