Name: ______________________________

Stat 515
Final Examination

Instructions:

- The total score is 100 points.
- There are six questions.
- Show **ALL** your work.
- Some questions have more than one parts. Check carefully to ensure that you did not miss any parts.
- Do not scratch on the line marked **Score** on the bottom of each page.
- You are allowed to use one 8.5x11(letter size) double-sided formula sheet and a calculator in this exam.

1. Let the cumulative distribution function(C.D.F.) of a continuous random variable Y be

$$F(y) = \begin{cases} 0 & y < 0, \\ \frac{y^2}{16} & 0 \le y < 4, \\ 1 & y \ge 4. \end{cases}$$

(7points) (a) Find the probability density function(p.d.f) of Y, $f(y)$.

(7points) (b) Find the expected value of Y, $E(Y)$.

(3points) (c) Find $P(1.5 \le Y \le 3)$.

(3points) (d) Find $P(Y \ge 1.5 \mid Y \le 3)$.

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2. A candy maker produces mints that have a label weight of 20.4 grams. Assume that the weights of these mints have a normal distribution with mean 21.37 and variance 0.16 (i.e., the weights of these mints $\sim N(21.37, 0.16)$). Let Y denote the weight of a single mint selected at random from the production line. Answer the following questions.

(5points) (a) What is the probability that Y is larger or equal to 22.07?

(5points) (b) What is the appropriate value for C such that a randomly chosen single mint has a weight less than C with probability 0.8531?

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3. The joint probability mass function of X and Y, $f(x,y)$, is given by

$$\begin{array}{lll} f(0,-1)=0, & f(0,0)=\frac{1}{3}, & f(0,1)=0, \\ f(1,-1)=\frac{1}{3}, & f(1,0)=0, & f(1,1)=\frac{1}{3}, \end{array}$$

and is zero otherwise.

(3points) (a) Find $f_X(x)$, the marginal distribution of X, and $f_Y(y)$, the marginal distribution of Y.

(4points) (b) Find the conditional distribution of Y given $\{X=1\}$.

(3points) (c) Are X and Y independent? Please support your answer with a proof or calculation.

(3points) (d) Compute $\mathbf{P}(X+Y>1)$ and $\mathbf{P}(XY=0)$.

(4points) (e) Compute $\mathbf{E}(X)$, $\mathbf{E}(Y)$, and $\mathbf{E}(XY)$. Hence compute $\mathrm{Cov}(X,Y)$.

(3points) (f) Do your findings in part (e) contradict your findings in part (c)? Why or why not?

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4. Let

$$f(x,y) = \begin{cases} 6xy^2, & x, y \in [0,1], \\ 0, & \text{otherwise.} \end{cases}.$$

(3points) (a) Show that $f(x, y)$ is a probability distribution function.

(4points) (b) Compute $f_X(x)$, the marginal distribution of X, and $f_Y(y)$, the marginal distribution of Y.

(2points) (c) Are X and Y independent? Please support your answer with a proof or calculation.

(7points) (d) Find $\mathbf{E}(3X - 4Y)$ and $\mathrm{Var}(\sqrt{18}X)$.

(4points) (e) Calculate $\mathrm{Var}(X\sqrt{18} - Y\sqrt{80})$.

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5. Let $X_1, X_2, ..., X_n$ independent, identically distributed random variables such that for $0 < p < 1$, we have that $P(X_i = 1) = p$ and $P(X_i = 0) = q = 1 - p$ for all $i = 1, ?, n$.

(5points) (a) Find the moment generating function for any random variable X_i.

(5points) (b) Find the moment generating function of $Y_n = X_1 + X_2 + \cdots + X_n$.

(5points) (c) What is the distribution of Y_n? Prove your claim using (b).

Score:____________

6. An airline company is considering a new policy of booking as many as 396 persons on an airplane that can seat only 370. Past studies have revealed that only 89% of the booked passengers actually arrive for the flight. The company decided to book 396 persons.

(7points) (a) Formulate the problem in a way that the Central Limit Theorem (CLT) can be applied and justify the use of the CLT.

(8points) (b) Estimate the probability that if the company books 396 persons, then not enough seats will be available.

Score:____________