1. Let the random variable $Z = X - Y$ represent the time spent at a service window where $(X, Y)$ has joint density

$$f(x, y) = \begin{cases} e^{-x}, & 0 \leq y \leq x \leq \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $E(Z)$. [2 pts]

(b) Is it highly likely that a randomly selected customer would spend more than 4 minutes at a service window? [2 pts]

2. Suppose $Y$ has a uniform distribution over the interval $(0, 1)$.

(a) Find the cumulative distribution function, $F(y)$. [1 pt]

(b) Show that $P(a \leq Y \leq a + b)$ for $a \geq 0, b \geq 0$ and $a + b \leq 1$ depends only on the value of $b$. [3 pts]

3. The joint probability mass function, $f(x, y)$, of $X$ and $Y$ is given by and is zero otherwise:

$$f(1, 1) = 0, \quad f(1, 2) = 0.05, \quad f(1, 3) = 0.05,$$
$$f(2, 1) = 0.1, \quad f(2, 2) = 0.45, \quad f(2, 3) = 0.1,$$
$$f(3, 1) = 0.1, \quad f(3, 2) = 0.1, \quad f(3, 3) = 0.05.$$

(a) Find the marginal distributions for $X$ and $Y$. [1 pt]

(b) Find the conditional distribution of $Y$ given $\{X = 1\}$. [2 pts]

(c) Are $X$ and $Y$ independent? Please support your answer with a proof or calculation. [1 pt]

(d) Compute $P(X + Y > 2)$. [1 pt]

4. Let $Y$ be a random variable with mean $\mu$ and moment generating function $\phi(t)$. Derive the mean of $U = aY + b$, for constants $a$ and $b$, using moment generating functions. [4 pts]

5. For random variables $Y_1, Y_2,$ and $Y_3$ let $E(Y_1) = 1, E(Y_2) = 2,$
$$E(Y_3) = 1, \ Var(Y_1) = 1, \ Var(Y_2) = 3, \ Var(Y_3) = 5, \ Cov(Y_1, Y_2) = 0.4, \ Cov(Y_1, Y_3) = 0.5, \text{ and } Cov(Y_2, Y_3) = 2.$$

(a) Find the variance of $U = Y_1 - 2Y_2 + Y_3$. [1 pt]

(b) If $W = 3Y_1 + Y_2$, find $\text{Cov}(U, W)$. [2 pts]