STAT 515-04

Quiz – 2015-12-08

Max time: 40 minutes. 20 points total. Show work for full credit. Clearly label and record your answers in the workbook provided.

1. Let the random variable $Z = X - Y$ represent the time spent at a service window where (X, Y) has joint density

$$f(x,y) = \begin{cases} e^{-x}, & 0 \le y \le x \le \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find $\mathbf{E}(Z)$. [2 pts]

   (b) Is it highly likely that a randomly selected customer would spend more than 4 minutes at a service window? [2 pts]

2. Suppose Y has a uniform distribution over the interval $(0, 1)$.

   (a) Find the cumulative distribution function, $F(y)$. [1 pt]

   (b) Show that $\mathbf{P}(a \le Y \le a + b)$ for $a \ge 0$, $b \ge 0$ and $a + b \le 1$ depends only on the value of b. [3 pts]

3. The joint probability mass function, $f(x, y)$, of X and Y is given by
and is zero otherwise.

$$\begin{array}{lll} f(1,1) = 0, & f(1,2) = 0.05, & f(1,3) = 0.05, \\ f(2,1) = 0.1, & f(2,2) = 0.45, & f(2,3) = 0.1, \\ f(3,1) = 0.1, & f(3,2) = 0.1, & f(3,3) = 0.05, \end{array}$$

   (a) Find the marginal distributions for X and Y. [1 pt]

   (b) Find the conditional distribution of Y given $\{X = 1\}$. [2 pts]

   (c) Are X and Y independent? Please support your answer with a proof or calculation. [1 pt]

   (d) Compute $\mathbf{P}(X + Y > 2)$. [1 pt]

4. Let Y be a random variable with mean μ and moment generating function $\psi(t)$. Derive the mean of $U = aY + b$, for constants a and b, using moment generating functions. [4 pts]

5. For random variables Y_1, Y_2, and Y_3 let $\mathbf{E}(Y_1) = 1$, $\mathbf{E}(Y_2) = 2$, $\mathbf{E}(Y_3) = 1$, $\text{Var}(Y_1) = 1$, $\text{Var}(Y_2) = 3$, $\text{Var}(Y_3) = 5$, $\text{Cov}(Y_1, Y_2) = 0.4$, $\text{Cov}(Y_1, Y_3) = 0.5$, and $\text{Cov}(Y_2, Y_3) = 2$.

   (a) Find the variance of $U = Y_1 - 2Y_2 + Y_3$. [1 pt]

   (b) If $W = 3Y_1 + Y_2$, find $\text{Cov}(U, W)$. [2 pts]