Worksheet 2.6 (Part 1) - Math 455

1. Suppose a drawer contains three red beads, four blue beads, and five green beads. Use a generating function to determine the number of ways to select six beads if one must select at least one red bead, an odd number of blue beads, and an even number of green beads. Assume that beads of the same color are indistinguishable, and that the order of selection is irrelevant.

2. Use a combinatorial argument to count the number of different five-card hands that can be dealt from a triple deck, then the number of five-card hands that can be dealt from a quadruple deck.

3. Use a generating function to determine the number of ways to select a hand of m cards from a triple deck, if there are n distinct cards in a single deck. Verify that your expression produces the correct answers when $n = 52$ and $m = 5$.

4. Suppose that an unlimited number of jelly beans is available in each of five different colors: red, green, yellow, white, and black.
   (a) How many ways are there to select twenty jelly beans?
   (b) How many ways are there to select twenty jelly beans if we must select at least two jelly beans of each color?

5. A catering company brings fifty identical hamburgers to a party with twenty guests.
   (a) How many ways can the hamburgers be divided among the guests, if none is left over?
   (b) How many ways can the hamburgers be divided among the guests, if every guest receives at least one hamburger, and none is left over?
   (c) Repeat these problems if there may be burgers left over.

Hints:

1. I can take one or two or three red beads and I can take 1 or 3 blue beads and 2 or 4 green beads. After writing a generating function representing this, what is the coefficient of x^6?

2. For a triple deck, what happens if no cards are repeated? Or one card is doubled? Or two cards are double? Or one card is triple? Or one card is tripled and another card is doubled?

3. Each card can be there zero, once, twice or three times. How can you represent that with a generating function? How can you find the coefficient for x^m in this expression? Use the same trick that we saw in class.

4. For part (b), instead of trying to write a new generating function and finding the coefficients, try to think about the problem in a different way. How many jelly beans do you really get to select?

5. For part (b), try thinking along the same lines as in the last question. For part (c), either add up all the possibilities and use an identity we have seen for binomial coefficients or decide to give all of the leftovers to your best friend who is very hungry.