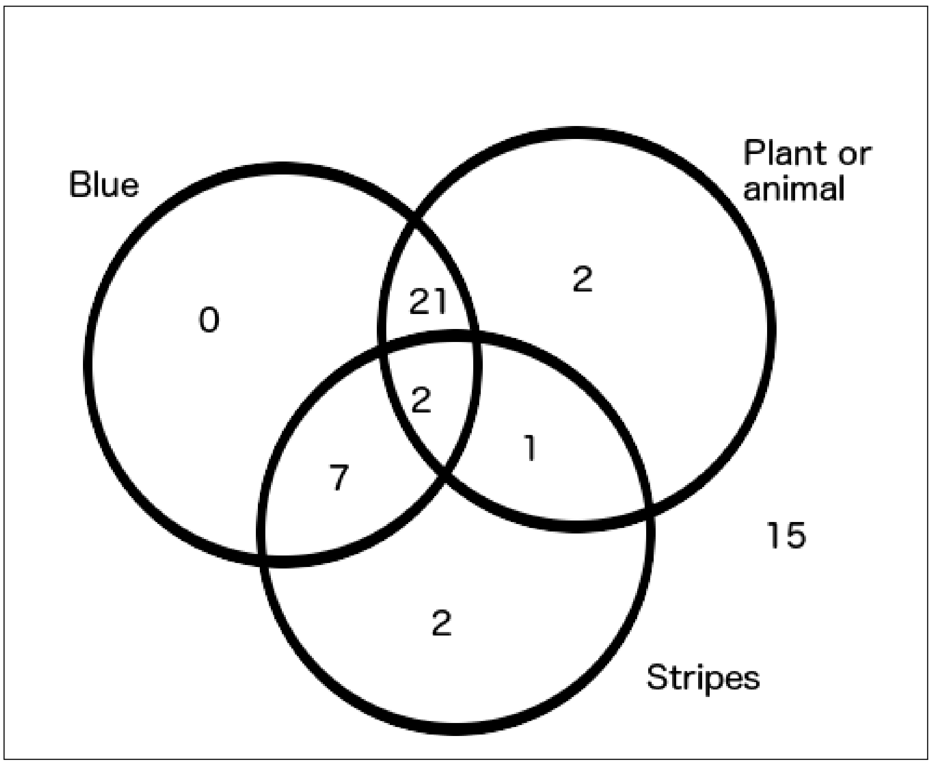
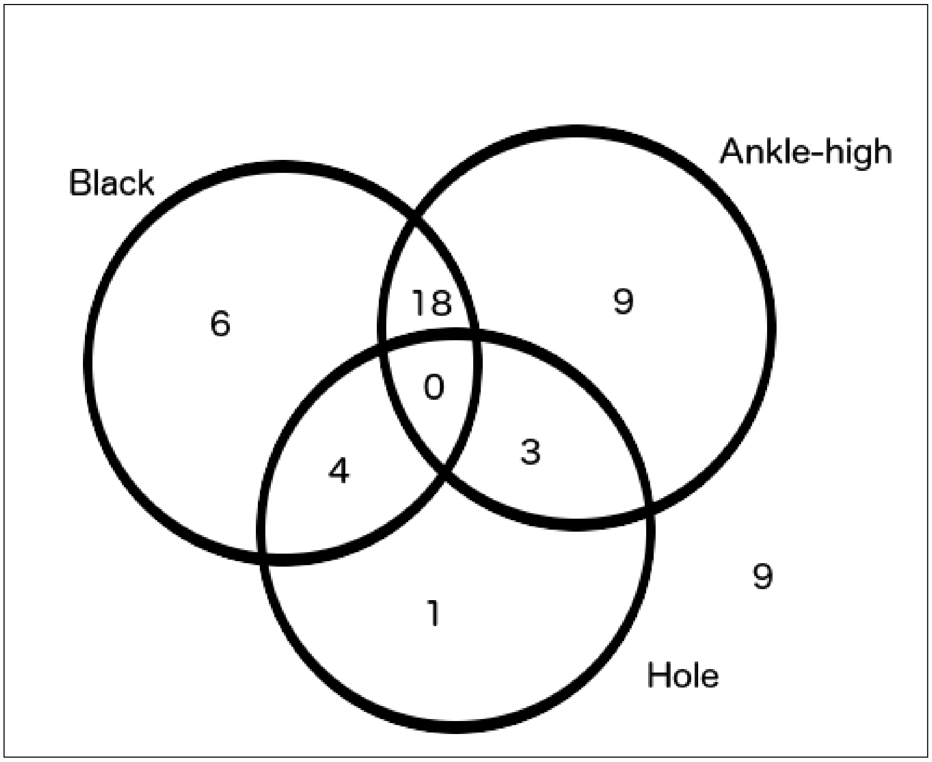
Worksheet 2.5 - Math 455

1. A noted vexillologist (Sheldon Cooper) tells you that 30 of the 50 US state flags have blue as a background color, twelve have stripes, 26 exhibit a plant or animal, nine have both blue in the background and stripes, 23 have both blue in the background and feature a plant or animal, and three have both stripes and a plant or animal. One of the flags in this last category (California) does not have any blue in the background. How many state flags have no blue in the background, and no stripes or animal featured.



Thus 15.

2. Suppose 50 socks lie in a drawer. Each one is either white or black, ankle-high or knee-high, and either has a hole or doesn't. 22 socks are white, four of these have a hole, and one of these four is knee-high. Ten white socks are knee-high, ten black socks are knee-high, and five-knee-high socks have a hole. Exactly three ankle-high socks have a hole. Use the principle of inclusion-exclusion to determine the number of black, ankle-high socks with no holes.



Thus 18.

3. Use the principle of inclusion-exclusion to determine the number of five-card hands drawn from a standard deck that contain at least one card from each of the four suits.

   There are $\binom{52}{5}$ hands in total. There are $52 - 13 = 39$ cards that are not hearts, so there are $\binom{39}{5}$ hands that contain no hearts. Thus, there are $4\binom{39}{5}$ hands not containing a specific suit. Note that in each case, it might be that we are missing more than one suits, so we are overcounting. There are $52 - 2 \cdot 13 = 26$ cards that are not hearts or diamonds, so there are $\binom{26}{5}$ hands that do not contain hearts or diamonds. Similarly, there are $\binom{4}{2}\binom{26}{5}$ hands that do not contain any of two specific suits. Again, we are overcounting. There are 13 cards that are not hearts, diamonds or clubs, so there are $\binom{13}{5}$ hands that do not contain any hearts, diamonds or clubs. Thus, there are $\binom{4}{3}\binom{13}{5}$ hands avoiding three specific suits. So we get that the cards that do not miss any suits are all the cards minus those that are missing suits, which by the inclusion-exclusion principle, gives us

$$\binom{52}{5} - \binom{4}{1}\binom{39}{5} + \binom{4}{2}\binom{26}{5} - \binom{4}{3}\binom{13}{5}.$$