Worksheet 2.4 - Math 455

1. Show that at any party with at least two people—can it really be a party with less than two people?—there must exist at least two people in the group who know the same number of other guests at the party. Assume that each pair of people at the party are either mutual friends or mutual strangers.

2. Wanting to get better at solving math problems, you decide to solve at least one math problem a day and at most 11 math problems per week for one year. Show that there must be a period of consecutive days in the year during which you will solve exactly 20 problems.

3. A positive integer is said to be *demonic* if it is written solely using 6's. For instance, 6, 66, 666 are all demonic. Prove that there exists a demonic number divisible by 2017.

4. Let S be any set of ten distinct integers chosen from $\{1, 2, \ldots, 99\}$. Show that S always contains two disjoint subsets S_1 and S_2 such that $\sum_{i \in S_1} i = \sum_{i \in S_2} i$.

Hints:

1. How could you represent this with a graph? This is essentially the same question as the one we had on the first worksheet that stated that any degree sequence (for a graph with at least two vertices) contains at least one pair of repeated entries.

2. Let a_i be the total number of problems you have solved since starting this regiment. Let $b_i = a_i + 20$. What can you say about these sequences of numbers? What are they bounded by? (It suffices to look at 8 weeks really!)

3. 2017 is prime. How many demonic numbers are there? How many different remainders are there if you divide demonic numbers by 2017?

4. What is the minimum value that $\sum_{i \in S_1} i$ can take? What is the maximum value it can take? How many subsets of S are there?