Worksheet 1.7 - Math 455

1. Find the minimum size of a maximal matching in C_n.

   If n is even, you can take every other edge

2. Let G be a bipartite graph. Show that G has a matching of size at least $\frac{|E(G)|}{\Delta(G)}$.

3. Let k be some fixed integer between 1 and n. Let G be some subgraph of $K_{n,n}$ with more than $(k-1)n$ edges. Prove that G has a matching of size at least k.

4. Draw a connected, 3-regular graph that has both a cut vertex and a perfect matching.

5. Determine how many different perfect matchings there are in $K_{n,n}$.

Hints:

1. Your answer should depend on n. How many edges in a row can you not have in a maximal matching?
2. Use König's theorem. How many edges can a single vertex cover? So how many vertices must there be in any edge cover?
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4. Look at figure 1.106 in the book. Can you modify that figure to obtain the desired outcome?
5. Start building a matching. To how many vertices can you match the first vertex? The second?