Worksheet 1.5.1 and 1.5.2 - Math 455

1. Find a planar representation of $K_{2,3}$.
2. Draw a planar graph in which every vertex has degree exactly 5.
3. Let G_1 and G_2 be two planar graphs with n vertices, q edges, and r regions. Must they be isomorphic?
4. How many regions are in a connected planar graph G of order 24 and regular degree 3?
5. Let G be a connected planar graph of order less than 12. Prove that $\delta(G) \leq 4$.
6. Prove that Euler's formula fails for disconnected graph.
7. Let G be of order $n \geq 11$. Show that at least one of G and $\bar{G}$ is nonplanar.
8. Show that there is no polyhedron with 5 vertices such that each pair of vertices is connected by an edge.
9. For a regular tetrahedron, take the midpoint of each of the 6 edges. Show that the solid whose vertices are those points is a regular octahedron.
10. Let F_k be the number of faces of a polyhedron P that are k-gons. For a simple polyhedron, i.e., a polyhedron where every vertex has degree 3, show that $3F_3 + 2F_4 + F_5 - F_7 - 2F_8 - 3F_9 - \ldots = 12$.

Hints:

1. You only need to move one vertex.
2. Think of Platonic solids.
3. Find a counterexample.
4. How many edges does such a graph have?
5. Adapt the proof of $\delta(G) \leq 5$.
6. Give an example.
7. What can you say about the number of edges in G and $\bar{G}$? Use the theorem that states that if G is planar, then the number of edges is at least $3n - 6$.
8. One way is to think about what is the shape of each face and use Euler's formula. Another way is to think about what the skeleton would look like.
9. You'll want to come up with actual coordinates for the vertices. To come up with good coordinates for the tetrahedron, find a tetrahedron within the regular cube with vertices $(\pm 1, \pm 1, \pm 1)$.
10. Relate the number of edges to the number of vertices.