SYLLABUS FOR MATH 797RT: REPRESENTATION THEORY

A. OBLOMKOV

1. Main Book

Our main source is "Introduction to Representation Theory", by Pavel Etingof, Oleg Golberg, Sebastian Hensel, Tiankai Liu, Alex Schwendner

It is available online: https://math.mit.edu/ etingof/repb.pdf The other sources will be provided by the instructor during the semester.

2. Material to be covered

We are planning to cover basics of representation theory as well as key methods. The key examples that are covered in class are finite groups, $GL_n(\mathbb{C})$ general linear group, S_n symmetric group, $GL_2(\mathbb{F}_q)$ and quiver algebras. If time permits, we cover theory of Soergel bimodules and applications to knot homology. We provide suggestions for further reading to explore the beyond the listed examples. It is expected that students in class prepare a presentation on a topic related to class discussion. A list of topics will offered in class.

Tentative class schedule

- Week 1: Reminder on Algebra: tensor product, ideals, algebras; Lie algebras, $\mathfrak{sl}(2)$.
- Week 2,3: Subrepresentations, the density theorem, filtrations; The Jordan-Hölder theorem.
- Week 3: Representation of finite groups: Maschke's theorem, characters, orthogonality of characters.
- Week 4: Character tables, tensor product of representations, Frobenius-Schur indicator.
- Week 5: Frobenius divisibility, virtual representations, induced representations.
- Week 6: Representations of S_n, the hook length formula.
- Week 7: Schur-Weyl duality, Schur polynomials.
- Week 8: Representations of $GL(\mathbb{F}_q)$.
- Week 9: Quiver representations, examples; Gabriel's theorem.
- Week 10: Introduction to categories.
- Week 11: Homological algebra.
- Week 12: Soergel bimodules and HOMFLY homology.
- Week 13: Presentations.

3. Grading

The final grade is based on the bi-weekly homework (0.6) and on the presentation at the end of semester (0.3) and class participation (0.1). The list of topics for the presentation

will be offered in class. Suggestions for students on the topics of presentations are also encouraged.

Grade distribution A: [0.8,1]; B: [0.6,0.8); C: [0.4,0.6); D: [0.2,0.4); F: [0,0.2).

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