Handout M523

Project #I

Hints:

(1.) First of all note that the sequence $a_n \geq 0$ for all $n \geq 1$. Therefore (by problem # 7 of section 2.1) we must have that the limit of a_n is also ≥ 0. Next, since $a_1 > 0$ and a_n increases the limit is > 0.

(2.) To prove (a) and find the limit follow the hint. Show that

$$a_{n+1} = 1 + \frac{1}{a_n} \qquad (*)$$

Next observe that if $\{a_n\}$ satisfy the relationship (*) and $a_n \to a$, then since $a \neq 0$ the limit a must satisfy the same equation, ie.

$$a = 1 + \frac{1}{a}.$$ Solve and use (1) to find a.