M624 HOMEWORK – SPRING 2025

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SET 1 - DUE 02/13/2025

From Chapter 3 (pp 145-146 -Section 5): 11, 12, 14a), 15, 16a), 23a).

SET 2 - DUE 02/27/2025

From Chapter 3 (pp 145-146 -Section 5): 14b), 16b), 19, 23b), 32

From Chapter 3 (pp 153): 4

Additional Questions (Chapter 3): After you had read carefully –as assigned in class– the proofs of Lemma 3.3 and Theorem 3.14 do the following

1) Explain why $J_F(y) - J_F(x) \leq \sum_{\{n\,:\,x<x_n\leq y\}} \alpha_n \leq F(y) - F(x)$ (proof of Lemma 3.13).

2) Show rigorously that $J_F(x) - F(x)$ is continuous (in proof of Lemma 3.13).

3) Rewrite explaining fully the proof of Theorem 3.14 in Chapter 3. Note you need to solve and use exercise 14 (given above in Chapter 3).

4) Justify the step (†) left in class in the proof of Lemma 3.9. More precisely. Justify why assuming that for $\delta > 0$ sufficiently small $m(E) > \delta$ we have that:

a) For an appropriate compact $E' \subseteq E$ we have that $m(E') \geq \delta$ and

b) How Lemma 1.2 (first Vitali-type covering Lemma) gives you that from a finite covering of E' (using compactness) by balls in $\mathcal{B}$ one can select a disjoint sub-collection of these balls, (call them $B_1, B_2, \ldots, B_{N_1}$) such that $\sum_{i=1}^{N_1} .m(B_i) \geq 3^{-d} m(E') \geq 3^{-d}\delta$. In other words, justify the two inequalities.