Vibrating Drum Head: 2D Wave Equation

\[ u(x, y, t) = \text{vertical displacement} \]

\[ \text{No horizontal motion} \]

Again horizontal component of Newton's Law gives that

\[ \text{ED = Boundary of } D, \text{ on } \partial D \text{ throughout } D. \]

Assume \( T \) is also constant in \( t \) (as before).

The vertical component in Newton's Law gives

\[ \mathbf{F} = \int T \cdot \frac{\partial u}{\partial n} \, ds = \iiint \rho u_{tt} \, dx \, dy \]

Recall \( \frac{\partial u}{\partial n} = \mathbf{V}u \cdot n \) (constant) m.a.

Then by Gauss-Green's theorem, we have

\[ \iiint \text{div} (T \mathbf{V}u) \, dx \, dy = \iiint \rho u_{tt} \, dx \, dy \]

\[ \Rightarrow \frac{1}{T} \Delta u = u_{tt} \quad (\text{recall } \text{div} (\mathbf{V}u) = \Delta u) \]

Additional homework #3

Set 1

| \begin{align*}
\rho \quad & u_{tt} = c^2 \Delta u \\ c = \sqrt{\frac{T}{\rho}}
\end{align*} |