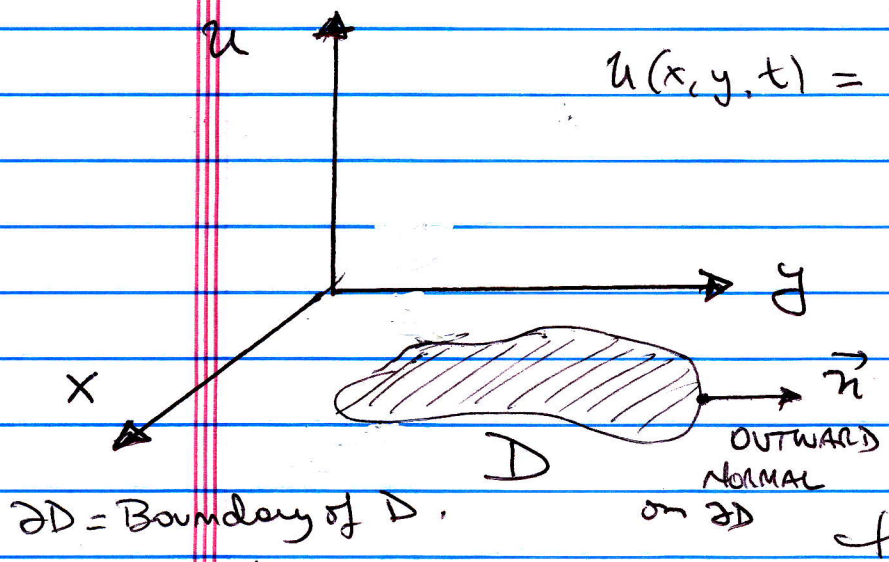
• Vibrating Drumhead : 2D Wave Equation



$u(x,y,t)$ = vertical displacement.
No horizontal motion

Again horizontal component of Newton's law gives that the tension T is constant throughout D.

∂D = Boundary of D.

Assume T is also constant in t (as before).

The vertical component in Newton's Law gives

$$(*)\quad \vec{F} = \int_{\partial D} T \cdot \frac{\partial u}{\partial \vec{n}}\, ds = \iint_D \rho\, u_{tt}\, dxdy$$

(the double integral is $m \cdot a$.)

• Recall $\frac{\partial u}{\partial \vec{n}} = \nabla u \cdot \vec{n}$ (T constant)

Then by Gauss-Green's theorem we have

$$\iint_D \operatorname{div}(T \nabla u)\, dxdy = \iint_D \rho\, u_{tt}\, dxdy$$

$$\Rightarrow \quad \frac{T}{\rho} \Delta u = u_{tt} \qquad (\text{recall } \operatorname{div}(\nabla u) = \Delta u)$$

Additional Hmw #3 Set 1

or $\boxed{u_{tt} = c^2 \Delta u}$ $\quad c = \sqrt{\frac{T}{\rho}}$