Remembering trig formulas

Which trig formulas should you remember? Certainly not all of them. Which ones, then? Here's some guidance. (I've taken some liberties with formatting MATHEMATICA's output below!)

1. Definitions of $\sin z$ and $\cos z$ in terms of power series.

2. Some other definitions are the same as in the real case, e.g., $\tan z = \dfrac{\sin z}{\cos z}$. So there's nothing really new to remember there.

3. Many trig identities are the same as in the real case, e.g., $\sin^2 z + \cos^2 z = 1$. So there's nothing really new to remember there.

4. Formulas for derivatives of trig functions, and of the inverse trig functions, are the same as in the real case, so there's nothing really new to remember there.

5. You should certainly know the identity $\exp(i\,z) = \cos z + i\,\sin z$. But that's easy to remember because it just generalizes the formula you already know in the case when z is real.

6. A number of the other trig formulas follow from the one for $\exp(i\,z)$, provided you also use it to get first a formula for $\exp(-\,i\,z)$. For example: $\sin z = \frac{1}{2\,i}\left(e^{i\,z} - e^{-i\,z}\right)$, and the corresponding formula for $\cos z$. But if you get stuck, MATHEMATICA is your friend:

```
   TrigToExp[Cos[z]]
Exp[I z]/2 + Exp[-I z]/2
```

7. You can quickly recover the definitions of sinh and cosh also by using MATHEMATICA's `TrigToExp`. Thus:

```
   TrigToExp[Sinh[z]]
E^z/2 - E^(-z)/2
```

8. Some other formulas to express trig functions in terms of functions of their real and imaginary parts are also quickly recovered by using `ComplexExpand` in MATHEMATICA. For example:

```
   ComplexExpand[Cosh[x + I y]]
Cos[y] Cosh[x] + I Sin[y] Sinh[x]
```

9. Problem the only "troublesome" formulas are the ones that express the inverse trig and inverse hyperbolic functions in terms of logs. Again MATHEMATICA's `TrigToExp` comes to your aid. For example:

```
   TrigToExp[ArcSin[z]]
-I Log[I z + Sqrt[1 - z^2]]

   TrigToExp[ArcTan[z]]
(I/2) (Log[1 - I*z] - (I/2)*Log[1 + I*z]
```

   Then to get the corresponding multi-valued functions, just replace the upper-case characters in"Arc" and "Log" with the corresponding lower-case characters so as to get "arc" and "log", etc.

   The formula just shown for differs just a bit from what you find in the text, namely,

$$\arctan z = \frac{i}{2}\,\log\left(\frac{i+z}{i-z}\right)$$

   but you can readily combine the difference of the two `Log` terms to get the log of a quotient as in the text's formula.