

Problem. Calculate $\text{Arcsin } \frac{5}{4}$, the principal value of $\arcsin \frac{5}{4}$.

Solution. By definition,

$$\text{Arcsin } \frac{5}{4} = -i \operatorname{Log} \left[i \frac{5}{4} + \left(1 - \left(\frac{5}{4} \right)^2 \right)^{1/2} \right] = -i \operatorname{Log} \left[i \frac{5}{4} + \left(-\frac{9}{16} \right)^{1/2} \right]$$

where $\left(-\frac{9}{16} \right)^{1/2}$ denotes the principal value of the square-root.

The principal square-root value is

$$\begin{aligned} PV \left(-\frac{9}{16} \right)^{1/2} &= \exp \left[\frac{1}{2} \operatorname{Log} \left(-\frac{9}{16} \right) \right] = \exp \left(\frac{1}{2} \left[\ln \left| -\frac{9}{16} \right| + i \operatorname{Arg} \left(-\frac{9}{16} \right) \right] \right) \\ &= \exp \left(\frac{1}{2} \left[\ln \frac{9}{16} + \pi i \right] \right) = \exp \left(\frac{1}{2} \ln \frac{9}{16} \right) \cdot \exp \left(\frac{\pi}{2} i \right) \\ &= \sqrt{\frac{9}{16}} i = \frac{3}{4} i. \end{aligned}$$

[Note that for arbitrary real $x < 0$, similarly $PV(x^{1/2}) = \sqrt{|x|} i$.]

Hence

$$\begin{aligned} \text{Arcsin } \frac{5}{4} &= -i \operatorname{Log} \left(\frac{5}{4} i + \frac{3}{4} i \right) = -i \operatorname{Log}(2i) \\ &= -i \left[\ln |2i| + i \operatorname{Arg}(2i) \right] = -i \left[\ln 2 + \frac{\pi}{2} i \right] \\ &= \frac{\pi}{2} - i \ln 2 \end{aligned}$$

Comment. That is the same result as obtained from MATHEMATICA if you evaluate the expression:

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ComplexExpand[ArcSin[5/4]]
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