Math 421 **Problem Set 9** **Fall, 2006**

Due: Monday, Dec. 11

Note: You may use MATHEMATICA to do basic computations—including factoring polynomials, finding derivatives, and calculating definite integrals over real intervals $[a, b]$—as well as algebraic simplifications involving real or complex numbers. Include printed output in such cases.

You may also want to use MATHEMATICA's `Series` and `Residue` functions to *check* the Taylor and Laurent series expansions and the residue calculations that you do with paper-and-pencil.

1. Do page 248, Exercise 10.
2. In the proof on page 246–247 of the Fundamental Theorem of Algebra (Theorem 6.19), why does the fact that $\lim_{|z|\to\infty} \dfrac{|a_{n-k}|}{|z^k|} = 0$ for each $k = 1, 2, \ldots, n$ imply that $\lim_{|z|\to\infty} \left(a_n + \dfrac{a_{n-1}}{z} + \dfrac{a_{n-2}}{z^2} + \cdots + \dfrac{a_0}{z^n}\right) = a_n$?
3. Do page 264, Exercise 2 (b).
4. Do page 264, Exercise 4 (a) and (b).
5. Do page 274, Exercise 2.
6. Do page 275, Exercise 10.
7. Do pages 283–284, Exercise 2 (b) and (l).
8. Do page 299, Exercise 1 (b) and (d).
9. Do page 300, Exercise 3 (b) and (f). Use the Residue Theorem where possible!
10. Do page 301, Exercise 8. Use the Residue Theorem where possible!