Math 421 **Problem Set 7** **Fall, 2006**

Due: Friday, Nov. 17

Note: You may use MATHEMATICA to do basic computations—including definite integrals—and algebraic simplifications involving real or complex numbers. (But when the computation involves *complex* numbers, know how to do it yourself with paper and pencil! Include printed output.

1. Do page 186, Exercise 1 by using the definition of $\cos z$ as the sum of a power series (Definition 5.5, page 176). Be sure to include all details.

2. Do page 186, Exercise 2.

3. Do page 187, Exercise 14 (a).

4. Derive identity (5-30), page 178: $\exp(i\,z) = \cos z + i\,\sin z$ for all z. Of course, do *not* use any identities that follow that one in the text (since many of them follow from this one!).

5. (c) Do page 192, Exercise 2 (c). Explicitly, what you must show is that the set $\{\, w : \tan w = z \,\}$ is the same as the set $\frac{i}{2}\,\log\left(\frac{i+z}{i-z}\right)$.

   (d) Do page 192, Exercise 2 (d). Explicitly, what you must show is: In the formula from (c), choose an arbitrary branch g of log to get a branch f of arctan. Then $f\,'(z) = \frac{1}{1+z^2}$ for all z at which $\frac{i+z}{i-z}$ is *not* a point of the branch cut of g (where g is either not defined or else defined but not differentiable). Note that you *not* need to be concerned at all as to what set that branch cut consist of.

6. Do page 197, Exercise 2.

7. (a) Do page 212, Exercise 3 (a); give both an exact value for the Riemann sum and a numeric value. Notice that the sample points c_k are the "midpoints" of the circular arcs having points of the subdivision of C as ends. Show the full setup with paper and pencil.

   (b) Do page 212, Exercise 3 (b).

   Extra credit: Again using "midpoints" of circular arcs as the sample points c_k, obtain numeric values for the Riemann sums that approximate this same contour integral for $n = 8, 16, 32, 64, \ldots$, stopping when finally your approximation agrees with the exact value of the integral to 5 decimal places. (You may use `MidpointSum` from `ContourIntegrals.nb`).

8. Do page 212–13, Exercise 6 (a) and (b).

9. Do page 213, Exercise 7 (d).

10. Do page 213, Exercise 14.