Math 421 **Problem Set 5** **Fall, 2006**

Due: Friday, Nov. 3

1. Do page 131, Exercise 2.
2. Do page 132, Exercise 10 (a). (*Hint:* Separate the real and imaginary terms.) Also, determine whether the series is absolutely convergent.
3. Do page 146, Exercise 2.
4. Do page 146, Exercise 5 (b) and (d).
5. Do page 153, Exercise 3 (a), (b), and (d).
6. Do page 154, Exercise 4.

   Hint: Differentiate $\displaystyle\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$; multiply the result by z.
7. Do page 154, Exercise 6.

Extra credit: Let $V = p + i\,q$ be the velocity field for an irrotational, incompressible steady-state fluid flow in a simply-connected domain D in the plane. We know that then V has on D a holomorphic "complex potential" $F = \phi + i\,\psi$ for which $F' = p - i\,q = \overline{V}$ and $V = \dfrac{\partial \phi}{\partial x} + i\,\dfrac{\partial \phi}{\partial y}$.

Fix a real constant K and consider the "level curve" $\psi(x, y) = K$ in the plane. Show that at each point (x, y) on the curve the vector $V(x, y)$ is tangent to the curve.

Hint: Calculate $\dfrac{dy}{dx}$ at a typical point (x, y) on the level curve. To do so, differentiate $\psi(x, y) = K$ implicitly with respect to x.

Interpretation: Since a fluid particle at a given point (x_0, y_0) has velocity vector $V(x_0, y_0)$, this result says the particle flows along the level curve $\psi(x, y) = \psi(x_0, y_0)$. Thus, the level curves of ψ are the **streamlines** of the flow.