1. (24 points) **Using Laplace transforms**, solve the initial-value problem

$$\frac{dy}{dt} + 4y = u_3(t), \quad y(0) = 5,$$

where $u_3(t)$ is the Heaviside function that "turns on" at $t = 3$.

3. (24 points) Consider the second-order equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 65\sin 2t.$$

a. Determine a particular solution to this differential equation.

b. Find the general solution to this differential equation.

4. (24 points) Solve the initial-value problem

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 0 & 4 \\ -1 & 4 \end{pmatrix} \mathbf{Y}, \quad \mathbf{Y}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$