Math 300.2 **Problem Set 2 Answers** **Fall, 2007**

1. [Exercise A.2.5 (4) first part.] Let (x, y) be an arbitrary ordered pair. Then

$$\begin{aligned}
(x, y) \in A \times (B \cup C) &\iff x \in A \;\&\; y \in B \cup C \\
&\iff (x \in A) \;\&\; (y \in B \text{ or } y \in C) \\
&\iff (x \in A \;\&\; y \in B) \text{ or } (x \in A \;\&\; y \in C) \\
&\iff (x, y) \in A \times B \text{ or } (x, y) \in A \times C \\
&\iff (x, y) \in (A \times B) \cup (A \times C). \qquad \square
\end{aligned}$$

The sets involved are illustrated in Figure 1. [The blue horizontal segment represents A, and the green and red vertical segments represent B and C, respectively. Then the green-shaded region represents $A \times B$ and the pink-shaded region represents $A \times C$, so that the entire shaded region represents $(A \times B) \cup (A \times C)$. Since the long vertical segment represents $B \cup C$, then the entire shaded region also represents $A \times (B \cup C)$.]

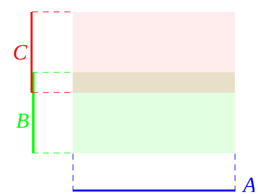

Figure 1: $A \times (B \cup C) = (A \times B) \cup (A \times C)$

2. [Exercise A.3.2 (4).] Let R be a relation in X. To say that $\Delta_X \subset R$ is to say that, for every $x \in X$, the ordered pair $(x, x) \in R$. In other words, $\Delta_X \subset R$ means that the relation R is reflexive!

3. [Exercise A.3.3 (3).] Assume first that $x \leq y$. Let $z \in X$ with $z < x$. We must deduce that $z < y$. Since $z \leq x$, by transitivity already $z \leq y$.

   To complete the proof that $z < y$, we must show that $z \neq y$. Just suppose $z = y$. Since $z < x$, this means $y < x$. Then $y \leq x$. Since also $x \leq y$, b antisymmetry, $x = y$. Then $z = x$, too. But then, since $x = z < y = x$, we have $x < x$. This means $x \neq x$, which is impossible.

   Conversely, assume that $z < x \implies z < y$ for all $z \in X$. We must deduce that $x \leq y$. Suppose not. In particular, $x \neq y$. Since $\leq$ *totally* orders X, then $y \leq x$. Since $x \neq y$, then $y < x$. Take $z = y$ in the assumed condition to obtain from $y < x$ that then $y < y$. Then $y \neq y$, which is impossible. $\square$

4. [Exercise B.1.7 (5).] First, the "where": $c_\Delta \colon I \times I \to \{0, 1\}$. Second, the "what": For $(x, y) \in I \times I$, by definition

$$c_\Delta(x, y) = \begin{cases} 1 & \text{if } (x, y) \in \Delta, \\ 0 & \text{if } (x, y) \notin \Delta. \end{cases}$$

In other words,

$$c_\Delta(x, y) = \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{if } x \neq y. \end{cases}$$

5. [Exercise B.1.7 (6) (a).] Let X and E be subsets of X. Note that c_S and c_E have the same domain X and the same codomain $\{0, 1\}$. Note further that, for all $x \in X$,

$$\begin{aligned} c_S(x) = 1 &\iff x \in S, \qquad & c_E(x) = 1 &\iff x \in E, \\ c_S(x) = 0 &\iff x \notin S, \qquad & c_E(x) = 0 &\iff x \notin E \end{aligned}$$

Assume first that $S = E$. Then for each $x \in X$ we have $x \in S \iff x \in E$, so that $c_S(x) = 1 \iff c_E(x) = 1$, and consequently $c_S(x) = c_E(x)$. Hence $c_S = c_E$.

Conversely, assume that $c_S = c_E$. For each $x \in X$ we have $c_S(x) = c_E(x)$, so that $c_S(x) = 1 \iff c_E(x) = 1$ and consequently $x \in S \iff x \in E$. Hence $S = E$. $\square$