Math 300.2 **Problem Set 5** **Fall, 2007**

Due: Friday, October 19, 1:00 p.m.
Leave in Liz's LGRT 1623D mailbox

1. Do Exercise 1.2.11 (b)—by induction, of course. Be careful to use *only* the recursive definition of addition and, if relevant, the associative law (a) for addition.

2. Now do Exercise 1.2.11 (c)—by induction, of course. Be careful to use *only* (b), the recursive definition of addition, and, if relevant, the associative law for addition.

3. Do Exercise 1.2.14 (1). (You may wish to defer starting this one until after the Tuesday, Oct. 16, class.)

4. Do Exercise 1.2.11 (2). Note that the Errata correct the definition of $m < n$ to say that there exists some $d \in \mathbb{N}^*$—instead of $d \in \mathbb{N}$—such that $m + d = n$. You may use any of the properties of addition in $\mathbb{N}$ that precede (2) in Exercise 1.2.11.

5. Do Exercise 1.2.22 (2).

6. Do Exercise 1.3.4 (4). You may use any of the properties of summation previously established in Section 1.3 along with the earlier formula for $\sum_{j=1}^{n} j$.

7. (a) Do Exercise 1.3.6 (3).
   (b) Do Exercise 1.3.7 (a).

8. Prove that, for all $m \in \mathbb{N}$, we have $m \not< m$. Recall our definition that $m < n$ means there exists some $k \in \mathbb{N}^*$ for which $m + k = n$. *Hint:* Use induction on m. You may use the Peano Postulates and the prior results that the associative law holds for addition in $\mathbb{N}$ and that $<$ is a transitive relation in $\mathbb{N}$.

   Remark: This result is one of the steps towards the "Gap Lemma".