Math 300.2 Practice problems for Exam 1 September 27, 2007

1. (a) Construct the truth table for the logical formula

$$\big((\neg P) \text{ or } Q\big) \iff (P \Longrightarrow Q)$$

   (b) According to the truth table, is that logical formula a tautology? Why or why not?

2. (a) State *in words*, as simply as possible, the meaning of:

$$(\forall m \in \mathbb{N})\ (\exists n \in \mathbb{N})\ (m < n)$$

   (b) Using words and/or symbols, form the negation (that is, the denial) of the statement shown in (a). Do *not* use 'not' or 'no', any synonym for 'not' or 'no', or any symbol for 'not' or 'no' in your answer!

   (c) Tell which one is true—the statement in (a) or its negation in (b)—and indicate why it is true.

3. If $X = \{1, 2, 3\}$ and if $Y = \{z, w\}$ with $z \neq w$, then:

   (a) List all the elements of $\mathcal{P}(X)$.

   (b) List all the elements of $X \times Y$.

   (c) How many *injective* functions are there from X to Y? Why?

4. Let A, B, and C be subsets of a set X. Complete each statement and then prove it:

   (a) According to one of the distributive laws, $A \cup (B \cap C) =$

   (b) According to one of **De Morgan's** Laws, $X \setminus (A \cup B) =$

5. Let A and B be subsets of a set X. Prove that $A \subset B$ if and only if $A \cup B = B$.

6. What is the definition that a relation R in a set X be **transitive**?

7. (a) Give an example of a function $f\colon \mathbb{Z} \to \mathbb{Z}$ that is surjective but not injective.

   (b) Let $f\colon A \to B$ and $g\colon B \to C$ be functions such that $g \circ f$ is surjective. Prove that g is surjective.

8. (a) Let $f\colon X \to Y$ be a function and let D and E be subsets of Y. Prove that

$$f^{-1}(D \cup E) \subset f^{-1}(D) \cup f^{-1}(E).$$

   (b) Must equality hold in (a)? If so, prove that it does. If not, give a counterexample.

9. Recall that a positive integer m is *odd* when it is not even, and then such $m = 2j - 1$ for some positive integer j.

   Use mathematical induction to prove that, for every $n = 1, 2, 3, \ldots$, the sum of the first n odd positive integers is n^2.

10. Recall that, by definition, an integer m is *divisible by* a positive integer d when there exists some integer k for which $m = d\,k$.

    Use mathematical induction to prove that, for every positive integer n,

$$5^{2n-1} + 1 \text{ is divisible by } 6.$$

11. State the Peano Postulates about 0, $\mathbb{N}$, and the successor function $\sigma\colon \mathbb{N} \to \mathbb{N}$.