Math 236 **Problem Set 8** **Spring, 2001**

Due date: Wed., Apr, 11 but #8 due Fri., Apr. 13

1. (a) Do page 120, Exercise 36.

   (b) Let T and $\vec{v_1}, \vec{v_2}, \ldots, \vec{v_m}$ be as in page 120, Exercise 37. Suppose $\ker(T) = \{\vec{0}\}$. Show that $T(\vec{v_1}), T(\vec{v_2}), \ldots, T(\vec{v_m})$ must be linearly independent, too.

2. Do page 120, Exercise 43.

3. Do page 132, Exercise 42.

4. Do page 192, Exercise 17.

5. Do page 221, Exercise 2. (Please refer to null spaces and column spaces of matrices rather than kernels and images—that is, use notation N and C instead of ker and im.)

6. (a) If $\overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_k}$ is an *orthonormal* basis of a subspace V of $\mathbb{R}^n$, then for every $\vec{x} \in \mathbb{R}^n$ we have the formula:

   $$\operatorname{proj}_V(\vec{x}) = (\vec{x} \cdot \overrightarrow{v_1})\overrightarrow{v_1} + (\vec{x} \cdot \overrightarrow{v_2})\overrightarrow{v_2} + \cdots + (\vec{x} \cdot \overrightarrow{v_k})\overrightarrow{v_k}$$

   Modify this formula for the more general case that the vectors in the basis $\overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_k}$ of V are *orthogonal* to one another (but do not necessarily all have length 1).

   (b) Use the result from (a) to do page 193, Exercise 28.

7. (Omitted from this set. See Problem Set 8.5.)

8. *(Counts as two problems!.)* In MATHEMATICA write a function `proj` that calculates projections *without* using orthonormal bases (and hence *without* using the Gram-Schmidt algorithm). For a list `{v1, v2, ..., vm}` of vectors in $\mathbb{R}^n$ and a vector `x` in $\mathbb{R}^n$, the result of `proj[{v1, v2, ..., vm}, x]` is to be the projection of `x` onto the span V of `{v1, v2, ..., vm}`.

   The method is to use the decomposition of the vector in question into its projections onto V and $V^\perp$.

   Of course, you will need first to form the given list of vectors into a matrix A. To form a basis of $C(A)$, note that $C(A) = R(A^T)$, the row space of the transpose of A. [For a matrix M, its *row space* $R(M)$ is the span of the *rows* of M—see page 132. The nonzero rows of $\operatorname{rref}(M)$ form a basis of $R(M)$.] For a matrix A, use the built-in function `NullSpace` to find a basis of $N(A)$.

   Test your function thoroughly. Details about validating your function `proj` will be found in notebook `Aboutproj.nb`.