

**Due: Friday, Feb. 16**

The original Problem 1 was already on Set 1; that has been deleted here and the rest renumbered. The original Problem 2, which is now Problem 1, was deferred from Set 1.

1. (Deferred from Set 1.) Do page 35, Exercise 24 and justify your answer.
2. Do page 36, Exercise 36. (*Hint:* See Exercises 34–35.)
3. Let  $\vec{u} = \begin{bmatrix} -2 \\ 3 \\ -9 \end{bmatrix}$  and  $\vec{v} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ . Calculate  $(8\vec{u} - 5\vec{v}) - 2(5\vec{u} + \vec{v})$ . (Use algebraic properties of vectors to minimize your work.) Then draw the standard representation of the resulting vector.

4. Let  $\vec{u} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$  and  $\vec{v} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ .

- (a) Determine all vectors  $\vec{x}$  that are parallel to  $\vec{v}$  and have the same length as  $\vec{u}$ . (Do this algebraically!) Tell which ones have the same direction as  $\vec{v}$  and which have the opposite direction.
  - (b) Draw a figure showing  $\vec{u}$ ,  $\vec{v}$ , and all the  $\vec{x}$  you determined.
5. In MATHEMATICA, use dot product (`Dot` or its abbreviation `.`) to define a function `perpQ` that takes as arguments two vectors having the same number of entries and returns `True` or `False` according as the two vectors are or are not orthogonal. Your `perpQ` should work no matter what the common dimension (length) of the vectors.

Test your `perpQ` by applying it to several examples and comparing MATHEMATICA's answers with what you obtain by paper-and-pencil calculation (aided by a calculator for the arithmetic, if you wish). Be sure to test with vectors of various dimensions  $n$ , not just for  $n = 2$  and  $n = 3$ , and for vector for which the result is `False` as well as those for which it is `True`.

Your `perpQ` should certainly be expected to work if the vectors are represented as ordinary lists (for example, `{1, 2, 3}`). For **extra credit**, make it work as well if the vectors are column vectors or row vectors. (*Hint:* Use the built-in function `Flatten`.)

6. (a) Use algebraic properties of vector operations and dot product, but do *not* use entries (coordinates) to prove that, for all vectors  $\vec{u}$  and  $\vec{v}$  in  $\mathbb{R}^n$ ,

$$\|\vec{u} - \vec{v}\|^2 = \|\vec{u}\|^2 - 2(\vec{u} \cdot \vec{v}) + \|\vec{v}\|^2.$$

- (b) Suppose  $\vec{u} \perp \vec{v}$ . From (a), we obtain the special case:

$$\|\vec{u} - \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2. \quad (*)$$

In  $\mathbb{R}^2$  draw a figure illustrating this—show all three vectors  $\vec{u}$ ,  $\vec{v}$ , and  $\vec{u} - \vec{v}$  on it and be sure to show  $\vec{u}$  perpendicular to  $\vec{v}$ . Then tell what familiar geometric theorem represents (\*).