## Speeding up and slowing down

Consider an object moving on a line, with velocity function $v(t)$ and acceleration function $a(t)$. To say the object is **speeding up** means that its speed is increasing; to say it is **slowing down** means that its speed is decreasing.

Now the speed at time $t$ is given by

$$\text{speed}(t) = |v(t)| = \begin{cases} v(t) & \text{if } v(t) > 0, \\ -v(t) & \text{if } v(t) < 0. \end{cases}$$

Hence its rate of change is

$$\begin{aligned} \frac{d}{dt}\,\text{speed} &= \begin{cases} v'(t) & \text{if } v(t) > 0, \\ -v'(t) & \text{if } v(t) < 0, \end{cases} \\ &= \begin{cases} a(t) & \text{if } v(t) > 0, \\ -a(t) & \text{if } v(t) < 0. \end{cases} \end{aligned}$$

Then

$$\begin{aligned} \frac{d}{dt}\,\text{speed} > 0 \quad & \text{if } v(t) > 0 \text{ and } a(t) > 0 \\ & \text{or if } v(t) < 0 \text{ and } -a(t) > 0, \end{aligned}$$

that is,

$$\begin{aligned} \frac{d}{dt}\,\text{speed} > 0 \quad & \text{if } v(t) > 0 \text{ and } a(t) > 0 \\ & \text{or if } v(t) < 0 \text{ and } a(t) < 0. \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{d}{dt}\,\text{speed} > 0 \quad & \text{if } v(t) > 0 \text{ and } a(t) < 0 \\ & \text{or if } v(t) < 0 \text{ and } -a(t) < 0, \end{aligned}$$

that is,

$$\begin{aligned} \frac{d}{dt}\,\text{speed} < 0 \quad & \text{if } v(t) > 0 \text{ and } a(t) < 0 \\ & \text{or if } v(t) < 0 \text{ and } a(t) > 0. \end{aligned}$$

Thus

$$\begin{aligned} \frac{d}{dt}\,\text{speed} > 0 \quad & \text{if } v(t) \text{ and } a(t) \text{ have the same sign,} \\ \frac{d}{dt}\,\text{speed} < 0 \quad & \text{if } v(t) \text{ and } a(t) \text{ have opposite signs.} \end{aligned}$$

So this means:

| | |
|---|---|
| object is speeding up | if $v(t)$ and $a(t)$ have the same sign, |
| object is slowing down | if $v(t)$ and $a(t)$ have opposite signs. |