Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer _______________________________________ **Section #** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 20, 2007**
4:00-6:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are seven (7) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any paper except this exam booklet and your 1-page "cheat sheet."
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator to do numerical calculations, show the setup leading to what you are calculating.
- Be prepared to show your UMass ID card when you **hand in your exam booklet *as you exit the room.***

QUESTION	PER CENT	SCORE
1	15	
2	14	
3	15	
4	15	
5	14	
6	14	
7	13	
TOTAL	100	

1. ($3 \times 5\% = 15\%$) Let $f(x) = 3 - x$.

   (a) Compute the Riemann sum R_4 that uses **4** subintervals and **right** endpoints for $f(x)$ on the interval $[1, 4]$.
   Show the sum's individual terms, not just the value of the sum.

   (b) Draw a figure that shows the rectangles corresponding to that Riemann sum R_4.

   (c) Find the exact value of $\int_1^4 f(x)\, dx$ by interpreting it in terms of area.

2. (14%) Use the **definition** of derivative as a limit to calculate $f'(2)$ for

$$f(x) = x^3 - 2x.$$

3. ($3\times 5\% = 15\%$) The function $f(x)$, defined for all $x \neq 0$, has ***second*** derivative

$$f''(x) = 2x - \frac{1}{x^2}.$$

(a) Determine at which x, if any, the graph of $f(x)$ has inflection points.

(b) Given its value $f'(-1) = 0$, find the ***first*** derivative $f'(x)$.

(c) Determine at which x, if any, the function $f(x)$ has:
(i) a local minimum; (ii) a local maximum.

4. $(3 \times 5\% = 15\%)$ Find the following limits. Justify each answer *without* using your calculator to estimate the limit or to graph the function.

   (a) $\displaystyle\lim_{x \to 0^-} \frac{1+x}{x}$

   (b) $\displaystyle\lim_{x \to 0} \frac{1-\cos x}{x\,\cos x}$

   (c) $\displaystyle\lim_{x \to 0^+} x^2 \ln(\sqrt[3]{x})$

5. (14%) The UMass Hadley Farm will construct a 675-square-foot pen to hold livestock. The pen will be rectangular, with one side along the bank of a straight stream, so only the other three sides need to be fenced. Perpendicular to the stream there will also be a length of fencing that divides the pen in half—to separate the llamas from the goats.

   What should the pen's dimensions be to minimize the total length of fencing?

   Indicate the meaning of the variables you use. Specify the domain on which you are minimizing the relevant function. Test that you really found a minimum.

6. (14%) Maxine Minuteman is building a snowwoman! She starts with a snowball and rolls it in the snow to make it grow bigger and bigger. If, 10 minutes into this project, the snowball's surface area is growing at 4 in^2/min and its radius is growing at 2 in/min, how fast at this time is the snowball's volume growing?

   You may use the formulas $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$ for the snowball's volume and surface area, respectively, in terms of its radius r.

7. Let $f(x) = 4x - e^x$.

(a) (4%) Explain why $f(x) = 0$ has **at least one** root in the interval $[0, 1]$. Use principles of calculus.

(b) (4%) Explain why $f(x) = 0$ has **at most one** root in $[0, 1]$. Use principles of calculus.

(c) (5%) Use Newton's Method with initial approximation x_1 to find the **third** approximation x_3 to the root of $f(x) = 0$ in the interval $[0, 1]$. Round your answer to **five** (5) decimal places.

This page left blank for additional work.