Name (Last, First) ______________________ **ID #** __________

Signature ______________________________

Professor ______________________ **Section #** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **Dec. 17, 2005**
1:30–3:30 p.m.

Instructions

- Turn off all cell phones!
- There are eight (8) questions.
- Do all work in this exam booklet. You may continue work to backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- In finding derivatives, show your work—do *not* just write down the answers your calculator gives you (although you may want to check them with the calculator).
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in this exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	14	
3	12	
4	12	
5	12	
6	12	
7	12	
8	10	
TOTAL	100	

1. ($4 \times 4\% = 16\%$) Use appropriate methods of calculus to find the *exact* values of the following limits. (Do this *without* using your calculator to estimate the limits.)

   (a) $\displaystyle \lim_{x \to 0} \frac{x + \sin x}{4\,x^3 - x}$

   (b) $\displaystyle \lim_{t \to \infty} \frac{(\ln t)^2}{t^2}$

   (c) $\displaystyle \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$ given that $f(5) = 4$ and $f\,'(5) = -95$.

   (d) $\displaystyle \lim_{x \to \infty} x^{1/x}$

2. Do *not* use numerical data (with decimals) or graphs obtained from your calculator to justify your answers. (But you may wish to check work with the calculator).

   (a) (7%) Find the intervals within $[0, 2\pi]$ on which

   $$f(\theta) = 2\cos\theta - \cos(2\theta)$$

   is ***increasing***. (*Hint:* You may wish to make use eventually of one or both of the identities $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$.)

Continued on next page →

Continuation of 2.

(b) (7%) Find the intervals on which

$$g(x) = e^{-\frac{1}{2}x^2}$$

is ***concave up***.

3. ($4 \times 3\% = 12\%$) A highly contagious and long-lasting strain of flu was accidently released when a lab containment unit leaked. The leak was fixed only after 107 people became sick with this flu strain. Each sick person is infecting 3 healthy people every 7 days. The sick people do not recover, and they continue to spread this flu. Let $S(t)$ be the total number of sick people t days after the leak.

   (a) Fill in the table below.

t	$S(t)$
0	107
7	428
14	1712
21	
28	
35	

   (b) Find a formula for $S(t)$.

If you could not find a formula for $S(t)$, *use* $S(t) = 100 \cdot (1.3)^{\,t}$ *in* (c)–(d)*:*

   (c) How fast is the number of sick people increasing per day 8 weeks after the leak?

   (d) Find a formula for the inverse function S^{-1}.

4. (12%) Find an equation for the tangent line to the curve having equation

$$x\,y^{3} + y - 23 = x$$

at its point where $y = 2$.

5. (a) (3%) Recalling that arctan means “inverse tangent”, simplify:

$$\tan(\arctan x) =$$

(b) (9%) Use the identity you obtained in (a) to derive the formula:

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

6. Perfect Paper Cup Company (PPCC) wants to make a cylindrical cup (with open top, of course, so people can drink out of it). The cup is to be made from exactly 22 in^2 of paper—not counting any paper wasted in cutting out the circular bottom, in attaching the bottom to the cylindrical piece, or in fastening together the ends of the rectangle that's rolled up to make the cylindrical piece.

   PPCC asks you to solve its **problem**: *What should the cup's dimensions be so that it will hold the maximum amount of liquid when it is filled to the brim?*

   (a) (5%) Obtain a formula for a function that should be maximized in order to solve the problem. What is the domain of that function? (*Note:* the volume V of a cylinder whose base is a circle of radius r and whose height is h is given by $V = \pi r^2 h$.)

   (b) (7%) Now solve PPCC's problem. Be sure to give reasons why your answer actually does maximize the function.

7. ($3 \times 4\% = 12\%$) Below you will obtain an approximation for $\sqrt{5}$. Note that $\sqrt{5}$ is a root r of the equation $f(x) = 0$ where $f(x) = x^2 - 5$.

   (a) Given an arbitrary approximation x_n of $r = \sqrt{5}$, derive Newton's formula for a better approximation by calculating the point x_{n+1} at which the linearization of $f(x)$ at x_n intersects the x-axis.

   (b) Starting with the initial approximation $x_1 = 2$ (because you know $\sqrt{4} = 2$ exactly), use the formula from (a) to obtain a better approximation x_2.

   (c) Now repeatedly use that formula to obtain successively better approximations x_3, x_4, etc., until you find an approximation correct to **5** decimal places—and indicate how you know that is when to stop. (Of course, do ***not*** use your calculator's value for $\sqrt{5}$ to decide when to stop!) Show all the successive approximations you obtain.

8. (10%) Travelling at 90 feet per second along a narrow road in your Hummer one dark night, you suddenly see a deer 500 feet ahead on the road. The deer, stunned by the glare of your headlights, freezes in place. You slam on the brakes, decelerating the car at a constant rate, until the car comes to a complete stop 10 seconds later. Are you able to stop your car before it hits the deer?

   Use methods of calculus! Clearly indicate the meaning of the variables used.

   Suggestion: Find the car's velocity function. Then find where the car stops.

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