DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 131 Spring 2005
FINAL EXAM

Your Name: ________________________________

Your Section Number: ________________________

Your Instructor's Name: ______________________

The exam consists of 7 questions. Each problem is worth the indicated number of points. You may use a calculator and a page of your own notes, but no books.

It is not sufficient to just write the answers. You must **show your work** to receive credit for each problem.

1.	(10)	______
2.	(10)	______
3.	(20)	______
4.	(10)	______
5.	(20)	______
6.	(15)	______
7.	(15)	______
TOTAL	(100)	

1. (10 pts) Consider the function

$$f(x) = \begin{cases} 2 - x & \text{if} \quad x \le 1 \\ ax^2 + bx + 1 & \text{if} \quad x > 1 \end{cases} .$$

Determine the values of a and b for which the function f is continuous and differentiable.

2. (10 pts) A snowball melts in such a way that its surface area decreases at a rate of 1 cm^2/min. Find the rate of change of the diameter decreases when the diameter is 10 cm. (Note: The surace area of a ball of radius r is $S(r) = 4\pi r^2$.)

3. (20 pts) Consider the function given by $e^y = x + y$.

   (a) (5) Use implicit differentiation to find dy/dx.

   (b) (5) Find the equation of the tangent to the graph of the function at the point $(2 - \ln 2, \ln 2)$.

Problem 3 continues next page

(c) (5) Compute d^2y/dx^2 in terms of x and y.

(d) (5) Determine whether the function $f(x)$ is concave up or concave down at the point $(2 - \ln 2, \ln 2)$.

4. (10 pts) Use the limit laws and L'Hospital rule to evaluate the following limits.

(a) (5 pts) $\lim_{x \to 0} \dfrac{\tan(6x)}{\sin(3x)}$

(b) (5 pts) $\lim_{x \to \infty} \left(1 + \dfrac{5}{x}\right)^x$

5. (20 pts) Let $f(x) = xe^x$.

   (a) (4 pts) Find all the vertical and horizontal asymptotes of $f(x)$.

   (b) (4 pts) Find $f'(x)$ and $f''(x)$.

   (c) (4 pts) Find all local maxima and minima. Justify your results carefully with either the First or Second derivative test.

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(d) (4 pts) Find all points of inflection. Justify your answer!

(e) (4 pts) Sketch the graph of $f(x)$, using a suitable scaling, labeling local maxima/minima and points of inflection.

6. (15 pts) A farmer wants to enclose with a wood fence a rectangular field with an area of 1000 square meter and then divide it in half with a metallic fence parallel to one side of the rectangle. The wood fence costs \$2 per meter and the metallic fence costs \$6 per meter. Find the dimension of the field which minimize the cost of material. Be sure to justify that your answer is indeed a minimum.

7. (15 pts) Let $f(x) = (7+x)\sqrt[3]{x-1}$.

   (a) (5 pts) Find the critical numbers of $f(x)$.

   (b) (10 pts) Compute the global maximum and minimum of the function $f(x)$ on the interval $[-2\,,\,2]$.