Name (Last, First) ______________________ **ID #** __________

Signature ______________________________

Lecturer ______________________ **Section #** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 19, 2006**
1:30-3:30 p.m.

Instructions

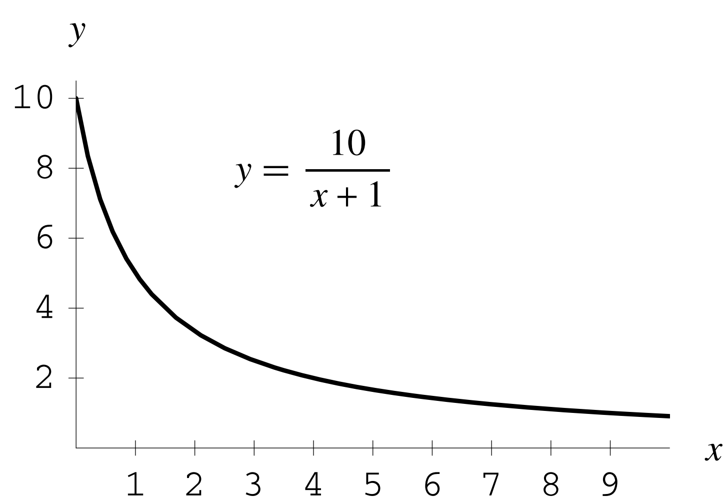
- **Turn off cell phones and watch alarms!** Put away cell phones, iPods, etc.
- There are eight (8) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any paper except this exam booklet and your 1-page "cheat sheet."
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator, show the setup for what you are calculating.
- Be prepared to show your UMass ID card when you **hand in your exam booklet** ***as you exit the room.***

QUESTION	PER CENT	SCORE
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
Free	4	4
TOTAL	100	

1. ($2 \times 6\% = 12\%$) Let $f(x) = \dfrac{10}{x+1}$.

(a) Approximate the definite integral $\displaystyle\int_1^9 f(x)\,dx$ by the Riemann sum that uses **4** subintervals and ***left*** endpoints as the sample points. Show the individual terms of the sum before you calculate the value of the sum.

(b) On the graph below, draw the rectangles that correspond to the Riemann sum in (a).



2. ($3 \times 4\% = 12\%$) The function $f(x)$ is defined, just for $x > 0$ by:

$$f(x) = x^2\, e^{-x}$$

(a) Calculate $f'(x)$ and $f''(x)$. (Show work!)

(b) Where is this function increasing? (Use methods of calculus.)

(c) Where is the graph of this function concave **up**ward? (Use methods of calculus.)

3. ($3 \times 4\% = 12\%$) Use appropriate methods of calculus to find the *exact* values of the following limits. (Do *not* use your calculator to estimate the limits.)

(a) $\displaystyle\lim_{x \to 0} \frac{x}{\arctan x}$

(b) $\displaystyle\lim_{x \to e} \frac{\ln x}{e^x}$

(c) $\displaystyle\lim_{x \to 0^+} (1 + x)^{1/\sqrt{x}}$

4. (12%) A particle moving along the x-axis has at each time $t > 0$ acceleration

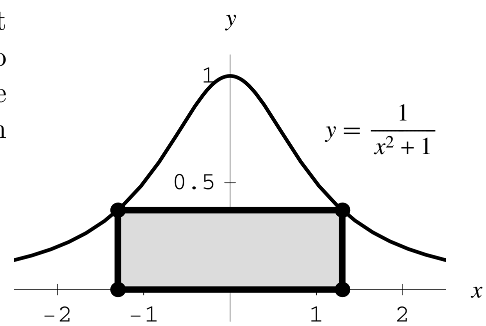
$$a(t) = 2t^2 + \frac{1}{t^4}.$$

At time $t = 1$ the particle is at the origin and has velocity 3.

(a) Find a formula for the particle's velocity v as a function of t.

(b) Find a formula for the particle's position x as a function of t.

5. (12%) Find the area of the largest rectangle that can be placed with two of its vertices on the x-axis and the other two of its vertices on the graph of $y = \dfrac{1}{x^2+1}$.



6. (12%) Find an equation for the tangent line to the curve

$$e^{(x\,y+1)} = y$$

at the point where $y = 1$.

7. ($3 \times 4\% = 12\%$) Let $f(x) = 2x + \sin x - 1$.

(a) Explain why $f(x) = 0$ has a root in $(0, 1)$. (Do *not* use your calculator to examine the graph.)

(b) Explain why $f(x) = 0$ has only one root. (Do *not* use your calculator.)

(c) Use Newton's method to approximate the root of $f(x) = 0$ to **three (3) decimal places** by choosing initial approximation $x_1 = 1$. Show, to at least 5 decimal places, all the intermediate approximations you find.

8. ($2 \times 6\% = 12\%$) The gravitational force exerted by Earth on an object at a distance r from Earth's center has strength $F(r)$ given by:

$$F(r) = \begin{cases} K\,r/R^3 & \text{if } 0 < r \leq R, \\ K/r^2 & \text{if } r > R. \end{cases}$$

Here R is the Earth's radius and K is another constant (determined by the mass of the object).

(a) Is the function $F(r)$ continuous at $r = R$? Why or why not?

(b) Is the function $F(r)$ differentiable at $r = R$? Why or why not? (*Hint:* Use the definition of derivative as a limit.)

This page left blank for additional work.