Name (Last, First) ______________________ **ID #** __________

Signature ______________________________________

Professor ______________________ **Section #** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **Dec. 18, 2004**
8:00–10:00 a.m.

Instructions

- Turn off all cell phones!
- There are eight (8) questions.
- Do all work in this exam booklet. You may continue work to backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in this exam booklet.

QUESTION	PER CENT	SCORE
1	14	
2	12	
3	12	
4	12	
5	14	
6	14	
7	12	
8	10	
TOTAL	100	

1. Let

$$f(x) = x^2 - 3\,x + 5.$$

(a) (6%) Use the **definition** of derivative to express $f\,'(x)$ for this particular function as the limit of a quotient (but do *not* yet evaluate this limit).

$f\,'(x) = \lim$

(b) (8%) Now evaluate that limit, thereby finding $f\,'(x)$.
Do *not* use L'Hospital's Rule!)

2. (12%) Find an equation of the tangent line to the curve with equation

$$e^{x\,y} = y$$

at the point where $y = 1$.

3. (12%) The population $P(t)$, in millions, of the land of Newtonia has been growing at a rate proportional to the population at time t (in years), so that

$$P(t) = A\,e^{k\,t}$$

for some constant A. If the population of Newtonia is currently 20 million and 10 years from now will reach 30 million, what is the value of k?

For the value of k give both an exact quantity and a decimal approximation.

4. ($2 \times 6\% = 12\%$) Let $f(x) = \dfrac{\sqrt{x^2+1}}{2\,x-1}$. By first determining the relevant limits—and *not* just using numerical evidence or plotting the graph on your calculator—find:

   (a) Equations of all horizontal asymptotes of the graph of f.

   (b) Equations of all vertical asymptotes of the graph of f.

5. (14%) You are going to construct a rectangular box with open top and square base. The material for the base costs \$2 per square foot, but the material for the sides costs \$1 per square foot. You can spend at most \$96, total, on these materials. What dimensions will allow the box to hold the greatest amount?

   Indicate the meaning of the variables you use. Specify the domain on which you are maximizing the relevant function. Test that you really found a maximum.

6. (14%) Let $f(x) = x\,e^{-x}$. Use methods of calculus to answer:

(a) Where is f increasing? decreasing?

(b) Where is f concave upward? concave downward?

(c) At what x, if any, does f have local maxima? local minima?

(d) At what x, if any, does f have inflection points?

7. ($3\times 4\% = 12\%$) Use relevant methods from calculus to find the following limits. Show work to justify your answers. (Using numerical evidence or examining graphs is *not* sufficient justification!)

   (a) $\displaystyle \lim_{x\to 0} \frac{\tan 2\,\pi\,x}{3\,\ln(1+x)}$

   (b) $\displaystyle \lim_{x\to 1} \frac{x^2-1}{2\,x-1}$

   (c) $\displaystyle \lim_{x\to 0^+} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

8. (10%) Driving along a dark, narrow road one night at 88 feet per second, you suddenly see a skunk on the road 350 feet ahead. The skunk, stunned by the glare of your headlights, freezes in place. You slam on the brakes, decelerating the car at a constant rate, until the car comes to a complete stop 8 seconds later. Are you able to stop your car before it hits the skunk?

   Use methods of calculus! Clearly indicate the meaning of the variables used.

   Suggestion: Find the car's velocity function. Then find where the car stops.

This page left blank for additional work.