Mathematics 131: Final, December 19, 2001

1) Consider the function

$$f(x) = \begin{cases} x^2 + x + k, & x \le 0 \\ ke^x, & 0 < x < 1 \\ kex, & x \ge 1. \end{cases}$$

A. Show that $f(x)$ is continuous for all values of k.

B. Determine the value of k so that the function $f(x)$ is differentiable for all x.

2) Find the points on the ellipse $2x^2 + y^2 = 1$ where the tangent line has slope 1.

3) If a (perfectly round) snowball melts so that its surface area decreases at a rate of 2 $\mathrm{cm}^2/\mathrm{min}$, find the rate at which the diameter decreases when the diameter is 5 cm. (Hint: surface area A of a ball $A = 4\pi r^2$ where r is the radius of the ball.)

4) Find the following limits

A. $\lim_{x\to 0} \frac{\sqrt{5+x}-\sqrt{5}}{x}$

B. $\lim_{x\to 0}[1 + \sin(2x)]^{1/x}$

5) Consider the function $f(x) = x^3 - 6x^2 + 9x - 3$ defined on the interval $[0, 4]$.

A. Find the critical points of $f(x)$ on $[0, 4]$ and decide if the critical points are local maxima or minima.

B. Find the absolute maximum and minimum values of $f(x)$ on $[0, 4]$.

C. Determine the intervals in the domain of $f(x)$ on which $f(x)$ is concave up.

6) Let $g(x) = (x^2 + 2x - 2)e^{-x}$ be defined on the interval $[-3, 4]$

A. The graph of $g(x)$ has two inflection points in $[-3, 4]$. Find both of them.

B. Find the two intervals in the domain of $g(x)$ on which $g(x)$ is decreasing.

7) A car starts 10 miles south of an intersection and travels north at 20 miles per hour. At the same time, a second car starts 20 miles east of the intersection and travels west at 10 miles per hour. Neither car stops at the intersection. At what time are the two cars as close together as possible, and how far are they then?

8) Find the dimensions of the rectangle with largest area, so that its base is on the x-axis and two of the other vertices are on $y = 8 - x^2$ and have positive y-coordinates.