Name (Last, First) ______________________ **ID #** __________

Signature ______________________________

Lecturer ______________________ **Section #** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 3** **November 28, 2007**
7:00-8:30 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	20	
3	15	
4	16	
5	16	
6	14	
Free	3	3
TOTAL	100	

1. (a) (4%) Find all critical numbers of the function $f(x) = x\sqrt{1 - x^2}$.

   (b) (12%) What are the absolute (that is, global) maximum value and the absolute (that is, global) minimum value of $f(x)$ on $[0, 1]$, and at which x in $[0, 1]$ are those values reached?

   (Use appropriate methods from calculus, *not* estimates obtained by graphing the function.)

2. ($4 \times 5\% = 20\%$) The **derivative** $f'(x)$ of a certain function $f(x)$ is given by:

$$f'(x) = x^4 - 3\,x^3$$

Use methods of calculus—*not* a graph plotted by your calculator—to answer the following *without* finding a formula for $f(x)$ itself. Show work to justify your answers!

(a) Where is $f(x)$ increasing? Where is it decreasing?

(b) Where is $f(x)$ concave upward? Where is it concave downward?

(c) At which x, if any, does f have an inflection point?

(d) At which x, if any, does f have a local maximum? A local minimum?

3. ($3 \times 5\% = 15\%$) Use appropriate methods of calculus to find the *exact* values of the following limits. (Do *not* use your calculator to estimate the limits.)

(a) $\displaystyle \lim_{x \to 1} \frac{\ln x}{\cos\left(\frac{\pi}{2} x\right)}$

(b) $\displaystyle \lim_{x \to 0} \frac{\sin x - x}{x^2}$

(c) $\displaystyle \lim_{x \to 0+} \left(\frac{1}{x}\right)^x$

4. (16%) The Red Sox are going to construct a wooden case to proudly display their World Series trophy. This box will have a square back and an open front, and it will have a volume of 4,000 cubic inches. What dimensions for the box will minimize the total amount of materials used for its sides and base?

   Follow this outline to find your solution:

   (a) (2%) Identify the variables involved (maybe draw a picture to help).

   (b) (4%) Determine what function (of a single variable) is to be minimized **and** on what domain.

   (c) (8%) Determine at what number that function takes its minimum value. Be sure to justify why the function actually does take its minimum there!

   (d) (2%) Answer the original question: what are the minimizing dimensions?

5. (16%) A bicyclist is riding directly east on a straight road at a steady rate of 25 ft/sec. A path running perpendicular to the road meets the road at a point in front of the bicyclist. A woman on the path who is north of the east-west road is jogging north along the path at a steady rate of 13 ft/sec.

   (a) (4%) Draw a diagram depicting the situation, carefully labeling all variable quantities.

   (b) (12%) How fast is the distance between the bicyclist and the jogger decreasing when the bicyclist is 24 feet from the north-south path and the jogger is 10 feet from the east-west road?

6. (a) (8%) Find the linearization $L(x) = \ldots$ of $\sqrt[4]{x}$ at $a = 16$.

(b) (6%) *Use this linearization* to approximate $\sqrt[4]{14.4}$. Give your answer as a decimal **rounded to 3 digits to the right of the decimal point**. (*Note:* The approximation you find need not be the same as the value your calculator gives for $\sqrt[4]{14.4}$.)

This page left blank for additional work.