Name (Last, First) _____ **ID #** _____

Signature _____

Lecturer _____ **Section #** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 2** **December 1, 2005**
6:00-7:30 p.m.

Instructions

- **Turn off all cell phones and watch alarms!**
  Put away cell phones, iPods, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	20	
2	20	
3	18	
4	14	
5	12	
6	16	
TOTAL	100	

1. (a) (6%) Find all critical numbers of the function $f(x) = x + \dfrac{4}{x^2}$.

(b) (14%) What are the (absolute) maximum value and the (absolute) minimum value of $f(x)$ on $[1/2, 4]$, and at which x in $[1/2, 4]$ are those values reached?

Use appropriate methods from calculus, *not* estimates obtained by graphing the function.

2. $(4 \times 5\% = 20\%)$ The **derivative** $f'(x)$ of a certain function $f(x)$ is given by:

$$f'(x) = 4x^3 - x^4$$

Use methods of calculus to answer the following *without* finding a formula for $f(x)$ itself. Show work to justify your answers!

(a) Where is $f(x)$ increasing? Where is it decreasing?

(b) Where is $f(x)$ concave upward? Where is it concave downward?

(c) At which x, if any, does f have an inflection point?

(d) At which x, if any, does f have a local maximum? A local minimum?

3. $(3 \times 6\% = 18\%)$ Use appropriate methods of calculus to find the *exact* values of the following limits. (Do this *without* using your calculator to estimate the limits.)

   (a) $\displaystyle\lim_{x \to 0} \frac{e^{-3x}}{x^2 - 2}$

   (b) $\displaystyle\lim_{x \to 1} \frac{\sin \pi x}{x^2 - x}$

   (c) $\displaystyle\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

4. (a) (5%) Find the linearization $L(x) = \ldots$ of $f(x) = \dfrac{1}{x}$ at $a = 2$.

(b) (5%) *Use this linearization* to approximate $1/2.1$. Give your answer as a decimal with at least 3 digits to the right of the decimal point. (*Note:* The approximation you find need not be the same as the value your calculator gives for $1/2.1$.)

(c) (4%) Is your approximation larger than or smaller than $1/2.1$? Why? For your answers, do ***not*** calculate the actual value by hand or by using your calculator. [*Hint:* Think about properties of the graph of $f(x)$.]

5. $(2 \times 6\% = 12\%)$

(a) Complete the sentence below so as to give a correct statement of the **Mean Value Theorem**:

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then:

[*complete the sentence*]

(b) Apply the Mean Value Theorem to $f(x) = \ln x$ on $[1, 2]$ to prove:

$$\ln 2 > \frac{1}{2}$$

6. (16%) A barge is traveling at a steady speed of 3 miles per hour along a straight riverbank when it passes a village there. Five minutes later a ferry leaves a dock directly across the river from the village and heads towards the village at 1.5 miles per hour. The river is 1/2 mile wide. How fast is the distance between the barge and the ferry changing five minutes after the ferry started crossing the river?

   (Identify the variables you use and what they represent. Indicate in terms of these variables what is given and what is to be found.)

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