Name (Last, First) _____ **ID #** _____

Signature _____

Lecturer _____ **Section #** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 3** **November 29, 2006**
7:00-8:30 p.m.

Instructions

- **Turn off all cell phones and watch alarms!**
  Put away cell phones, iPods, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	20	
2	16	
3	12	
4	16	
5	16	
6	16	
Free	4	4
TOTAL	100	

1. ($4 \times 5\% = 20\%$) The **derivative** $f'(x)$ of a certain function $f(x)$ is given by:

$$f'(x) = 3x^2 - x^3$$

Use methods of calculus—*not* a graph plotted by your calculator—to answer the following *without* finding a formula for $f(x)$ itself. Show work to justify your answers!

(a) Where is $f(x)$ increasing? Where is it decreasing?

(b) Where is $f(x)$ concave upward? Where is it concave downward?

(c) At which x, if any, does f have an inflection point?

(d) At which x, if any, does f have a local maximum? A local minimum?

2. $(4 \times 4\% = 16\%)$ Use appropriate methods of calculus to find the *exact* values of the following limits. (Do *not* use your calculator to estimate the limits.)

(a) $\displaystyle\lim_{x \to 1/2} \frac{\ln(2x)}{\sin(2\pi x)}$

(b) $\displaystyle\lim_{x \to 0} \frac{\cos 4x - \cos 3x}{x^2}$

(c) $\displaystyle\lim_{x \to 0} \frac{e^x - 1}{x - 1}$

(d) $\displaystyle\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

3. (a) (6%) Find the linearization $L(x) = \ldots$ of $\sqrt[5]{x}$ at $a = 32$.

(b) (6%) *Use this linearization* to approximate $\sqrt[5]{30}$. Give your answer as a decimal with **at least 3 digits to the right of the decimal point**. (*Note:* The approximation you find need not be the same as the value your calculator gives for $\sqrt[5]{30}$.)

4. (a) (4%) Find all critical numbers of the function $f(x) = x + \dfrac{6}{x+2}$.

(b) (12%) What are the absolute (that is, global) maximum value and the absolute (that is, global) minimum value of $f(x)$ on $[0, 1]$, and at which x in $[0, 1]$ are those values reached?

Use appropriate methods from calculus, *not* estimates obtained by graphing the function.

5. A rocket is rising vertically at a constant speed of 300 ft/sec. A cyclist is traveling along a straight road at a steady 20 ft/sec. When the cyclist passes under the rocket, it is 600 feet above her.

   (a) (4%) Draw a diagram depicting the situation, carefully labeling all variable quantities.

   (b) (12%) How fast is the distance between the observer and the rocket increasing **1 second later**? (Begin by stating clearly, in terms of the variables you used, what rates are given and what rate is to be found.) You may give your final answer in exact form or round it to two decimal places.

6. (16%) An engineer is designing a closed cylindrical can that will contain $56\pi\,\text{cm}^3$ of soup. What should the dimensions of the can be in order to minimize its cost? Ignore any metal needed to join the top and bottom to the side. (You may round your final answer to two decimal places.)

   Follow this outline to find your solution:

   (a) Identify the variables involved (maybe draw a picture to help).

   (b) Determine what function (of a single variable) that is to be minimized **and** on what domain.

   (c) Determine at what number that function takes its minimum value. Be sure to justify why the function actually does take its minimum there!

   (d) Answer the original question: what dimensions minimize the can's cost?

This page left blank for additional work.