Name (Last, First) _____ **ID #** _____

Signature _____

Professor _____ **Section #** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 2** **Dec. 1, 2004**
6:30–8:00 p.m.

Instructions

- There are six (6) questions.
- Do all work in this exam booklet. If your work won't fit in the space provided, clearly indicate where it is continued. (You may use backs of pages and the blank page at the end.)
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	20	
2	20	
3	20	
4	16	
5	16	
6	8	
TOTAL	100	

1. ($4 \times 5\% = 20\%$) A particle is moving along an axis according to the law of motion

$$s = t^3 - 12\,t^2 + 36\,t$$

where its position s is in meters and the time t is in seconds.

(a) (5%) Find the velocity function $v(t)$ for this motion.

(b) (5%) Find the acceleration function $a(t)$ for this motion.

(c) (5%) When is the particle speeding up? When is it slowing down?

(d) (5%) When is the particle moving forward? When is it moving backward?

2. (20%) What are the (absolute) maximum value and the (absolute) minimum value of

$$f(x) = x - \ln x$$

for $\frac{1}{e} \leq x \leq e$, and at which x in $\left[\frac{1}{e}, e\right]$ are those values reached?

Use an appropriate method from calculus, *not* the graph of the function, to obtain your answer. (But you may wish to sketch the graph on your calculator to see that the answer makes sense.)

3. $(4 \times 5\% = 20\%)$ The **derivative** $f'(x)$ of a certain function $f(x)$ is given by:

$$f'(x) = x^2 - 2\,x$$

Use methods of calculus to answer the following *without* finding a formula for $f(x)$ itself. Show work to justify your answers!

(a) Where is $f(x)$ increasing? Where is it decreasing?

(b) Where is $f(x)$ concave upward? Where is it concave downward?

(c) At which x, if any, does f have an inflection point?

(d) At which x, if any, does f have a local maximum? A local minimum?

4. (a) (8%) Find the linearization $L(x) = \ldots$ of the function $f(x) = \sqrt[3]{x}$ at the number $a = 8$. (Show your work!)

(b) (8%) Use this linearization to approximate $\sqrt[3]{8.1}$. Show your work. Give your answer either as an exact fraction or else as a decimal with all the digits that your calculator shows. (*Note:* The approximation you find need not be the same as the value your calculator gives for $\sqrt[3]{8.1}$!)

5. (16%) At 3:00 p. m., the B43 bus starts at Smith College and travels due East at 30 miles per hour. At the same time of 3:00 p.m., the R38 bus leaves Amherst and travels due South to Mt. Holyoke at 36 miles per hour. Amherst is 15 miles due East of Smith. How fast is the distance between the two buses changing at 3:20 p.m.?

   (Identify the variables you use and what they represent. Indicate in terms of these variables what is given and what is to be found.)

6. (a) (4%) Fill in the blanks and complete the sentence below so as to give a correct statement of the **Mean Value Theorem**:

If f is _____ on the closed interval $[a, b]$

and if f is _____ on the open interval (a, b),

then: [*complete the sentence*]

(b) (4%) Is there a function f such that

$$f(0) = 1, \quad f(1) = 3, \quad \text{and} \quad f'(x) \leq 1 \text{ for all } x\,?$$

You must justify your answer by applying the Mean Value Theorem as you stated it above

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