

Name (Last, First) _____ ID # _____

Signature _____

Lecturer _____ Section # _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131

Exam 1

October 3, 2007
7:00-8:30 p.m.

Instructions

- **Turn off all cell phones and watch alarms!**
Put away cell phones, iPods, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page “cheat sheet” that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	16	
4	18	
5	16	
6	15	
Free	3	3
TOTAL	100	

1. ($2 \times 8\% = 16\%$) At time t , in seconds, a particle moving along a line has coordinate $s(t) = t^2 + 4t - 2$, in meters.

(a) Calculate the particle's **average velocity** over the time interval $[3, 3.1]$.
Give your answer as a single number with appropriate units.

(b) Calculate the particle's **average velocity** over the time interval $[3, 3+\Delta t]$
where $\Delta t > 0$. Simplify your answer.

2. Consider $f(x) = \frac{3x^2 + 1}{x^2 - 9}$.

(a) (8%) By calculating relevant limits, determine the equations of all vertical asymptotes of the graph of $f(x)$. If there are none, say so.

(b) (8%) By calculating relevant limits, determine the equations of all horizontal asymptotes of the graph of $f(x)$. If there are none, say so.

3. Let $f(x) = \frac{1}{2x+3}$.

- (a) (10%) Find the slope of the tangent line to the graph of $y = f(x)$ at $x = 1$ by computing a relevant limit.

- (b) (6%) Write an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 1$. [If you were unable to complete (a), you may use m to denote the slope of $f(x)$ at $x = 1$.]

4. ($3 \times 6\% = 18\%$) Determine the following limits. Justify your answers (**without** reference to any graphs or numbers you might use your calculator to find).

(a) $\lim_{x \rightarrow 0} \frac{2x^2 + 1}{x^2 - 1}$

(b) $\lim_{\theta \rightarrow \pi} \theta^2 \cos \theta$

(c) $\lim_{t \rightarrow 1} \frac{e^t + 1}{e^t - 1}$

5. (a) (12%) Use the **definition** of derivative to calculate $\frac{dy}{dx}$ if $y = x^3 - x$.

(b) (4%) Use your result in (a) to determine $\left. \frac{dy}{dx} \right|_{x=5}$.

6. The function f is defined by

$$f(x) = \begin{cases} kx & \text{if } x < 2, \\ 3x^2 & \text{if } 2 \leq x, \end{cases}$$

where k is a constant.

(a) (8%) Find a value of the constant k that makes f continuous at $x = 2$.

(b) (7%) For that value of k , is f differentiable at $x = 2$? Why or why not?

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