Name (Last, First) ______________________ **ID #** __________

Signature ______________________________

Lecturer ______________________ **Section #** ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 1** **October 6, 2005**
6:00-7:30 p.m.

Instructions

- **Turn off all cell phones and watch alarms!**
  Put away cell phones, iPods, etc.
- There are eight (8) questions.
- Do all work in this exam booklet. You may continue work to backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	20	
4	16	
5	16	
6	16	
Extra credit	6	
TOTAL	106	

1. $(4 \times 4\% = 16\%)$ Let $f(x) = \sqrt{\dfrac{1-2\,x}{x}}$.

   (a) Determine the domain of f.

   (b) Derive a formula for the inverse function f^{-1} of f.

   (c) What is $f\left(f^{-1}\left(\frac{2}{\pi}\right)\right)$? (An exact value, *not* a numeric approximation!)

   (d) If $g(y) = y^2$ for all y, then what is $(g \circ f)(3\,x)$?

2. ($2 \times 8\% = 16\%$) During a bad storm, the height of a river was measured every two hours, starting at midnight. Here is data obtained for the height h, in feet above sea level, as a function of the elapsed time t, in hours, since midnight:

t	0	2	4	6	**8**	10
h	6.0	9.9	11.6	11.1	**8.4**	3.5

(a) Using only this data, calculate the *best* estimate you can for the **rate**, in feet per hour, at which the river's height was changing at 8 A.M. Do *not* round your answer; do indicate how you obtained it. (Do *not* try to fit a formula to this data!)

(b) Actually, the rate at time $t = 8$ at which h was changing with respect to t was exactly -1.75 feet per hour. Write an equation for the tangent line to the graph of the function h of t at the point where $t = 8$. (The variables are time t and height h.)

3. ($4 \times 5\% = 20\%$) Find each of the following limits. If the limit does not exist, say so. (Do this *without* using your calculator. You may want to use your calculator to confirm your answers.)

   (a) $\displaystyle \lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$

   (b) $\displaystyle \lim_{s \to \infty} \frac{7\,s^3 - 4\,s + 2}{3 + s^2 - 6\,s^3}$

   (c) $\displaystyle \lim_{x \to 0} \frac{2\,x}{\sqrt{x + 4} - 2}$

   (d) $\displaystyle \lim_{t \to 0} \frac{e^t - 3}{5 - \cos t}$

4. ($2 \times 8\% = 16\%$) Use the **definition** of the derivative in terms of limits to find:

(a) $f'(3)$ where $f(x) = x^2 - 4x + 5$.

(b) $g'(x)$ where $g(x) = \dfrac{1}{3 - x}$.

5. (16%) Through decay, Chemical Q loses one-**third** of its mass every 7 years. Twenty-five years ago, a container of Chemical Q was buried. The EPA has just dug it up and found 1,000 grams remaining. How much Chemical Q was originally buried?

   (Carefully identify the variables you use!)

6. ($2 \times 8\% = 16\%$) A magnetic field is induced by electrical current moving through a wire. The strength B of the magnetic field at a distance r from the center of a wire is given by:

$$B(r) = \begin{cases} B_0 \frac{r}{r_0} & \text{if } r \leq r_0, \\ B_0 \frac{r_0}{r} & \text{if } r > r_0 \end{cases}$$

for certain non-zero constants B_0 and r_0.

(a) Is the function $B(r)$ continuous at r_0? Why or why not?

(b) Is the function $B(r)$ differentiable at r_0? Why or why not?

Extra credit (6%) Let

$$f(x) = 3\,x - 5.$$

Illustrate the precise, (ϵ, δ), definition that

$$\lim_{x \to 2} f(x) = 1$$

by finding *algebraically* a suitable δ for

$$\epsilon = 0.01.$$

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