

DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 131 Spring 2005
EXAM 1

Your Name: _____

Your Section Number: _____

Your Instructor's Name: _____

The exam consists of 5 questions. Each problem is worth the indicated number of points. On this exam you may use a calculator and a page of your own notes, but no books.

It is not sufficient to just write the answers. You must *show your work* to receive credit for each problem.

1. (20) _____

2. (20) _____

3. (20) _____

4. (20) _____

5. (20) _____

TOTAL (100)

1. Compute the following limits.

$$(a) \quad (5) \quad \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{x^2}$$

$$(b) \quad (5) \quad \lim_{x \rightarrow 2} \frac{x^3 - 2x^2 + x + 2}{x + 2}$$

$$(c) \quad (5) \quad \lim_{x \rightarrow \infty} \frac{4x^3 - 2x^2 + 5}{2x^3 + 3x^2 - x - 3}$$

$$(d) \quad (5) \quad \lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x - 3}$$

2. (a) (6) State the definition of "The function $f(x)$ is continuous at the point $x = a$ ".

- (b) (8) Let $f(x)$ be the function given by

$$f(x) = \begin{cases} x^2 + c^2 & \text{if } x < 1 \\ 2cx & \text{if } x \geq 1 \end{cases}$$

Find the values of c for which the function f is continuous at $x = 1$.

- (c) (6) For the value of c found in (b), is the the function $f(x)$ differentiable?
(Explain your answer)

3. For the following functions compute the derivative **using the definition of the derivative**.

(a) (10) $f(x) = x^2 - x$.

(b) (10) $g(x) = \frac{1}{1+3x}$.

4. Let $f(x) = \frac{x^2}{x^2 - 1}$.

(a) (7) Find the vertical and horizontal asymptotes of $f(x)$.

(b) (8) Find the points at which the tangent line to the graph of $f(x)$ is horizontal.

(c) (5) Give a graph of $f(x)$.

5. Let $f(x) = x(2\sqrt{x} - 6)$.

(a) (7) Find $f'(x)$.

(b) (7) Find the equation of the tangent line to the graph of f at the point $(4, -8)$.

(c) (6) Find the point at which the tangent line to the graph of f is parallel to the line $4y - 8x + 5 = 0$.

